KANGAROO MATH THAILAND

Pre-Ecolier

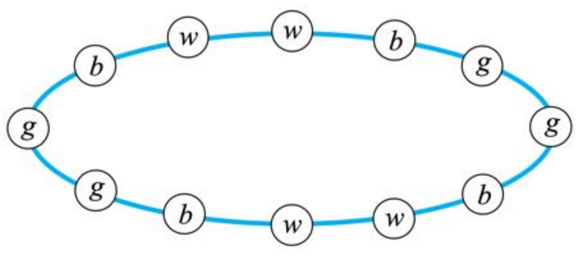
PRE-ECOLIER SOLUTION

KANGAROO MATH THAILAND
2019

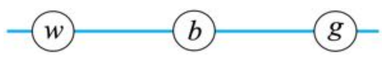
PRE-ECOLIER SOLUTION

1. (D)
   Option (A) is wrong because 8 is bigger than 7
   Option (B) is wrong because 9 is bigger than 7
   Option (C) is wrong because 7 is not less than 7
   Option (E) is wrong because 10 is bigger than 7

2. (C)



b = black
g = grey
w = white

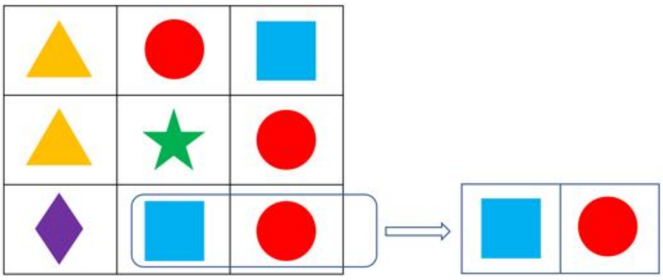


3. (C)
   60 → 5 and 10
   5 and 2
   ──────── −
   0 and 8

Therefore

$$\begin{array}{r} 60 \\ 52 \\ \hline 08 \end{array} -$$

4. (E)

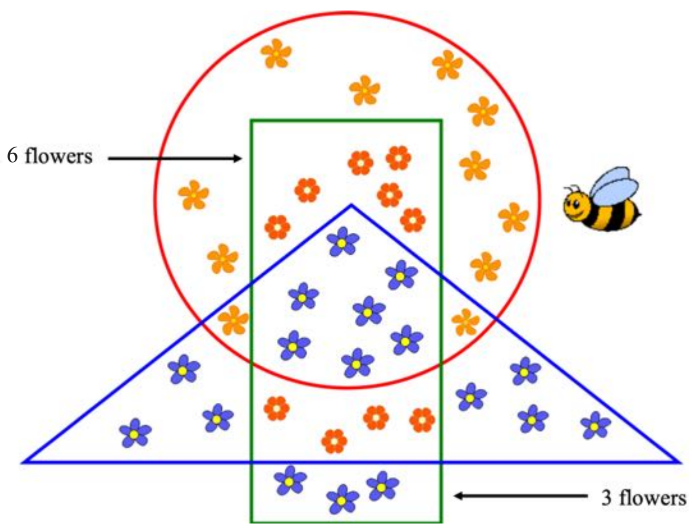


5. (B)

6. (C)

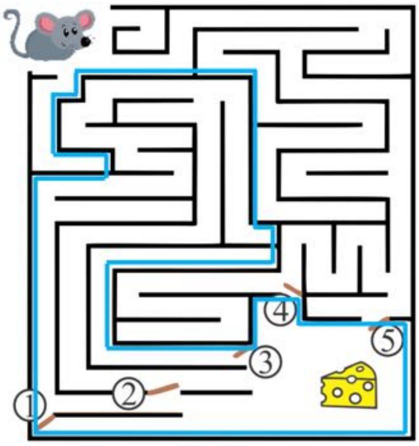
5 pairs: socks numbered 1, 2, 3, 5 and 7

7. (A)

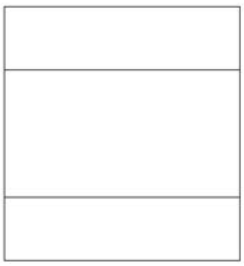


8. (C)
One apple costs $6 \div 2 = 3$ cents
One pear costs $8 \div 2 = 4$ cents
One apple and one pear cost $3 + 4 = 7$ cents

9. (E)

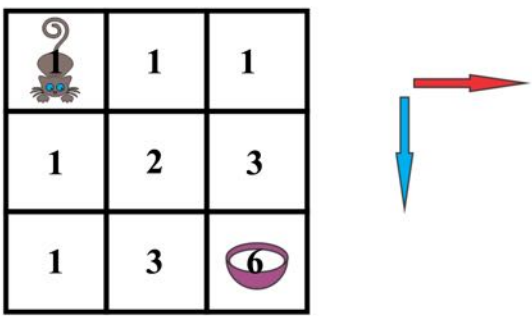


10. (B)
The sheet will be cut in this manner. Hence 3 pieces of paper.

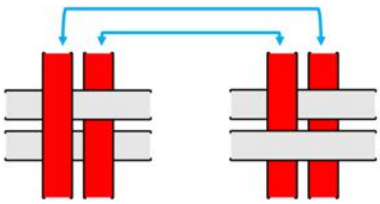


11. (A)
From top view, only card 5 is square. Hence the order is 5 – 2 – 3 – 1 – 4.

12. (E)



13. (B)



14. (B)
Figure (A) has 18 squares
Figure (B) has 16 squares
Figure (C) has 18 squares
Figure (D) has 18 squares
Figure (E) has 18 squares

15. (E)
The length of the tile is 4 m. Hence the length of the side with the question mark is
1 + 4 + 1 + 1 + 4 + 1 = 12 m

16. (D)
From 06 : 00 in the morning to 11 : 00 at night → (12 – 6) + 11 = 17 hours.
Therefore 17 – 2 – 3 – 7 = 5 hours

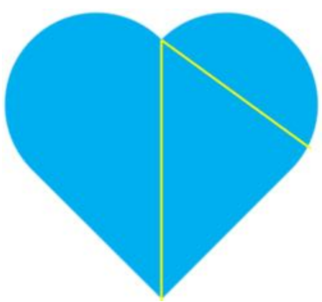
17. (D)

Sheep

Cows + 8

Hence = 8
Therefore total number of animals = 3 × 8 = 24.

18. (A)



19. (D)
If all camels are dromedary then there will be 10 humps. Since there are 14 humps, the numbers of bactrian camels is
$14 - 10 = 4 \rightarrow$ there is 4 extra humps
Hence 4 camels are bactrian.

20. (B)
Anni, Asia and Elli must have collected
Anni = 1 nut
Asia = 2 nuts
Elli = 7 – 1 – 2 = 4 nuts

21. (C)
The length of flagpole = 80 – 20 = 60 cm. Half the length of the flagpole = 60 ÷ 2 = 30 cm.
Hence the height of the sand castle is 30 + 20 = 50 cm.

22. (D)
Initially $b - w - g - w - g - b - w - b - g$
Hence $w - w - g - w - g - w - w - w - g$
Hence $w - w - b - w - b - w - w - w - b$
Hence $g - g - b - g - b - g - g - g - b$

23. (A)
The square of four cells can either

13	14
18	19

or

14	15
19	20

24. (C)
Amalia can use the first machine once and second machine twice.
3 red + 1 white
→ 2 red + 4 white (first machine)
→ 4 red + 3 white (second machine)
→ 6 red + 2 white (second machine)
Hence in total 8 tokens.

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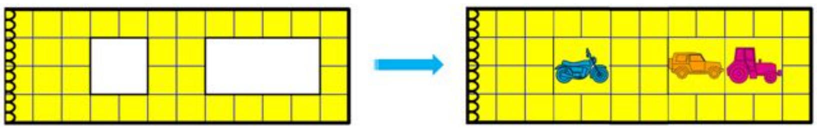
1. (E)
   D is higher than B then E, C and A accordingly.

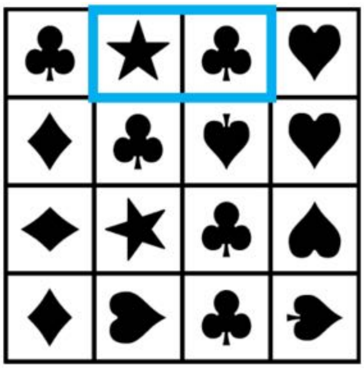
2. (C)
   $12 = 10 + 2 = 2 \times 5 + 2$
   Hence, 2 bars and 2 dots.

3. (A)
   Yesterday was Sunday. Today is Monday. Tomorrow will be Tuesday

4. (D)
   When the book is closed, the cover looks like



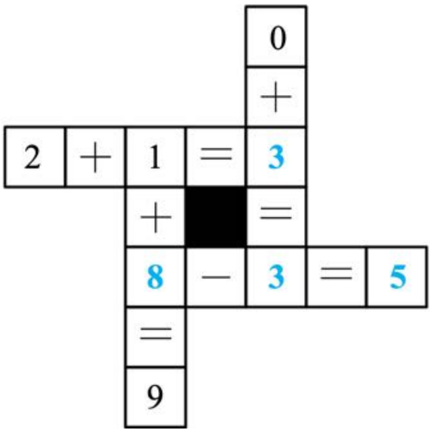
5. (A)



6. (A)
Look at the part where the paths are overlapped. The first footstep must be covered by the later footstep.

7. (D)
Pia has 10 connected sticks. Figure (D) needs 12 connected sticks.

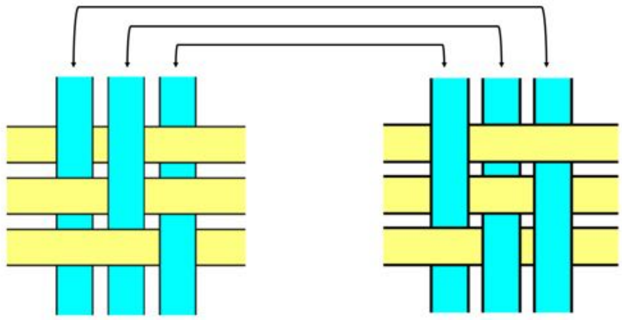
8. (B)



9. (B)
The photo on top needs 4 pins. The other $7 - 1 = 6$ photos needs 2 pins each. Hence in total $4 + 6 \times 2 = 4 + 12 = 16$ pins.

10. (C)
He can only get the first 3 shapes.

11. (C)



12. (E)
The weight of a dog is less than 12 kg but is more than $20 \div 2 = 10$ kg. Hence it weighs 11 kg.

13. (B)
16 blue is traded to 1 blue and 5 red.
5 red is traded to 1 red and 10 green.
Hence in the end she has 10 green marbles (together with 1 blue and 1 red marble).

14. (A)
He can use $920 + 1 = 921$ **or** $921 + 0 = 921$. Hence either 0 **or** 1 in the question mark box.

15. (D)
The weight of only water if it is full $= 400 - 100 = 300$ g. Half of water $= 300 \div 2 = 150$ g.
Hence the weight of a half-full glass of water $= 100 + 150 = 250$ g.

16. (D)
Adding altogether:
2 apples + 2 pears + 2 banana cost $5 + 7 + 10 - 22$ cents.
Hence, 1 apple + 1 pear + 1 banana cost $22 \div 2 = 11$ cents.

17. (E)

3○ = 12 → ○ = 4
★ + ♡ = 15 – 4 = 11
since ★ + 2♡ = 16
♡ = 16 – 11 = 5
★ = 11 – 5 = 6

18. (C)
10×10 picture needs 10 white squares on each side and 4 white squares at corners. Hence in total $4 \times 10 + 4 = 40 + 4 = 44$.

19. (B)
The digits 5 appears in the order:
5, 15, 25, 35, 45, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59
The 17^{th} digits 5 to come is 65.
Hence the maximum number of pages the book could have is 64.

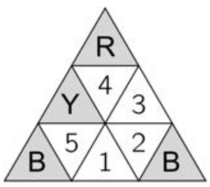
20. (E)
The two horizontal lines altogether has the length equal to 36 + 28 = 64 m. The vertical line has the length

$$\begin{aligned} 20 + 6 - (6 \div 2 + 8 \div 2) &= 26 - (3 + 4) \\ &= 26 - 7 \\ &= 19 \end{aligned}$$

Hence in total 64 + 19 = 83 m.

21. (B)
10 are not cows → 15 – 10 = 5 cows.
8 are not cats → 18 – 8 = 7 cats.
Therefore, number of Kangaroos = 15 – 5 – 7 = 3.

22. (E)



(Three blue triangeles already determined)
4 must be B
5 must be R
1 must be Y
2 must be R
3 must be Y

23. (B)
Alek's and Edek's statements contradict.
– One of them must be false.
– Bartlek's, Czarek's and Darek's statements must be true.
Hence Bartek has eaten a cookie.

24. (D)
Figure 1: each towel needs 2 pegs.
Figure 2: each towel needs 1 peg and 1 extra peg.

$$\begin{aligned} 58 &= 44 + 14 \\ &= (2 \times 22) + (13 + 1) \end{aligned}$$

where 22 is the number of towels in figure 1 and 13 is the number of towels in figure. 2.

KANGAROO MATH THAILAND

Benjamin

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1. (B)
   She added the 2 eyes and the hairs.

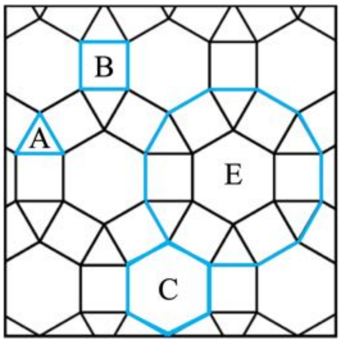
2. (C)
   $17 = 3 \times 5 + 2$
   $\therefore$ 3 bars and 2 dots.

3. (C)
   The next number using the same digits would be 2091. However, it is not impossible as time on the digital clock.
   $\therefore$ The next number is 2109 or 21:09.

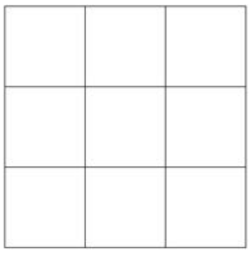
4. (E)
   Total number of children = 14 + 12 = 26.
   $\therefore$ Half of children go for a walk = 26/2 = 13.
   $\therefore$ 12 boys and 13 – 1 = 1 girl.

5. (E)
(A) is wrong as 2 and 5 dots must be opposite.
Similarly, for (B) : 3 and 4
(C) : 3 and 4
(D) : 1 and 6

6. (D)
There is no figure (D)



7. (D)
There are four 2×2 squares at the corners of the figure. There are another four 2×2 squares in



Hence, in total there are 8.

8. (C)
The 6 smallest odd natural number are 1, 3, 5, 7, 9 and 11.
Note that: *odd + odd + odd must be odd.*
However, 20 is an even number.

9. (B)
$60 - 36 = 24$ years
Each Kangaroo's age increased by 2 years.
$\therefore$ Number of Kangaroos $= \dfrac{24}{2} = 12.$

10. (A)
Building (B), (C), (D) and (E) each has a total surface area of $8 \text{ x } 4 + 4 \text{ x } 2 = 40$ small squares.
However, building (A) has a total surface area of $40 + 2 = 42$ small squares.

11. (C)
Assume the covered digits are all zeros.
The sum of the three number is
$243 + 107 + 026 = 376$
$\therefore 826 - 376 = 450$
The covered digits are at tens and hundreds. Hence the covered digits are 4 and 5.
Therefore $4 + 5 = 9$

12. (C)
If she is not hungry, in 9 days she would eat $5 \text{ x } 9 = 45$ spiders.
$\therefore$ Per day, if she is hungry she eats extra $10 - 5 = 5$.
$\therefore \dfrac{60 - 45}{5} = \dfrac{15}{5} = 3$ days.

13. (C)
Figure (C) cannot be drawn without lifting your pencil off the page.

14. (B)
Fraction of the square that is black
(A) $\dfrac{2}{4} = \dfrac{1}{2}$
(B) $\dfrac{5}{9}$
(C) $\dfrac{8}{16} = \dfrac{1}{2}$
(D) $\dfrac{13}{25}$
(E) $\dfrac{18}{36} = \dfrac{1}{2}$
The largest fraction is $\dfrac{5}{9}$.

15. (A)
The white triangle at the base would have the side of 2 cm.
$\therefore$ Side the largest thriangle is $1 + 2 + 2 = 5$ m
$\therefore$ Perimeter: $3 \times 5 = 15$ m

16. (C)
Doing backwards

	Dogs	Cats	Mice
End	10	10	10
	10	15	5
Start	16	9	5

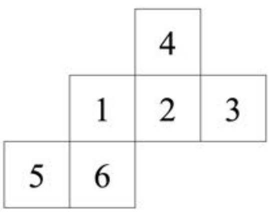
17. (B)
28 = 1 + 2 + 3 + 4 + 5 + 6 + 7 → The tower has 7 blocks at the base standing.
The height of the tower is
= 2 + 1 + 2 + 1 + 2 + 1 + 2
= 11 cm

18. (C)

The square sheet will be cut as shown in the figure.
∴ 9 pieces of paper.

19. (A)
Bob is not wearing a hat → Alex wears a hat.
Also, Bob is not wearing a hat → Carl wears a hat
Hence both Alex and Carl wear hats.
Note: Concept used is $p \Rightarrow q \equiv \overline{q} \Rightarrow \overline{p}$.

20. (D)



1: front 2: right 3: behind
4: top 5: left 6: bottom

21. (C)
The product is a multiple of 5, 10 and 15.
∴ The least possible is 30.
∴ The other three numbers are 6, 3 and 2.
Hence the smallest possible sum is $6 + 3 + 2 + 5 + 10 + 15 = 41$.

22. (E)
Let b and w (in g) be the weight of each black and white bead respectively.
$2b = 2w + 6 \quad \rightarrow b - w = 3 \quad (1)$
$3b + w = 1b + 30 \quad \rightarrow 2b + w = 30 \quad (2)$

Adding equation (1) and (2), we get:
$3b = 33$
$b = 11$
From equation (1), we can calculate w:
$b - w = 3 \rightarrow w = 8$
Hence the total weight (in gram) of these nine beads is $6 \times 11 + 3 \times 8 = 66 + 24 = 90$.

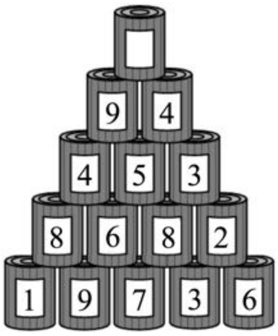
23. (D)
Statement (A) and (D) cannot be both true.
∴ (B), (C) and (E) are all true.
∴ Robert has 3 daughter and 2 sons.
Hence (D) is false.

24. (C)
Let a, b, c, d, e and f be the numbers in the circles in this order.
∴ a, b and c are consecutive whole numbers
∴ Exactly one of them is divisible by 3.
Since $d = 3c$ so d must be divisible by 3 which makes $e = d + 2$ and $f = 2e = 2d + 4$ **NOT** divisible by 3. Hence exactly 2 numbers are divisible by 3.

25. (B)
If the 2 black squares are front, the top must be both grey squares.

26. (B)
If each cousin is in 2 pictures $\rightarrow 8 \times 2 = 16$.
If each cousin is in 3 pictures $\rightarrow 8 \times 3 = 24$.
In each picture, there are exactly 5 cousins. Multiple of 5 in between 16 and 24 is 20.
Therefore, number of selfies $= \dfrac{20}{5} = 4$.

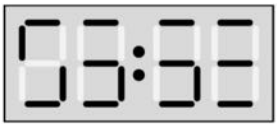
27. (D)
The pyramid at first must be



The top can must be 25 – (3 + 8 + 2 + 3 + 4) = 5
Therefore, Willi scores 8 + 4 + 9 + 5 = 26 points.

28. (A)
The 2 segments that do not work are

The time now is 23:47. 3 hours 45 minutes later, the time must be 03:32. Yet the 2 segments do not work. It must show



29. (D)
The way to use the white cubes:
8 → at the corners of the $4 \times 4 \times 4$ cube.
$12 \times 2 = 24$ → at the edges
All the white cubes are at the sides of the $4 \times 4 \times 4$ cube.
Number of white squares
$8 \times 3 = 24$ (corners)
$24 \times 2 = 48$ (edges)
Total = 24 + 48 = 72 white squares.
Total surface = $6 \times 16 = 96$ squares.
Therefore, fraction equals to $\frac{72}{96} = \frac{3}{4}$

30. (C)

Every time he uses 1st and 2nd machine, the number of tokens increases by 3 and 2, respectively. Let a and b be the number of times he used 1st and 2nd machine, respectively.

$a + b = 11 \quad \rightarrow \quad a + b = 11 \qquad (1)$

$3a + 2b = 31 - 4 \quad \rightarrow \quad 3a + 2b = 27 \qquad (2)$

Substracting equation (2) with 2 × equation (1), we get:

$a = 5$

$b = 6$

The number of red tokens now:

$5 \times 4 - 6 \times 1 = 20 - 6 = 14.$

KANGAROO MATH THAILAND

Cadet

CADET SOLUTION

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2019

CADET SOLUTION

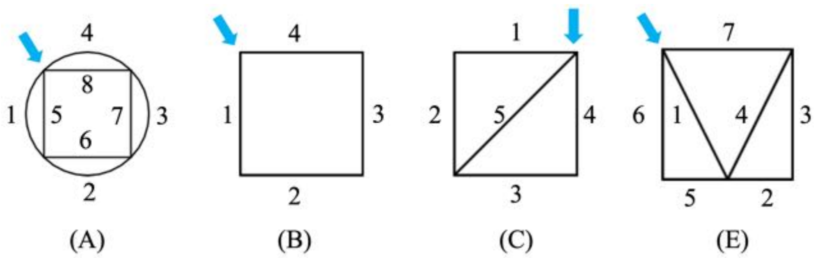
1. (E)
   2, 10, 34 and 58 are all even numbers.

2. (E)
   $\frac{10}{4} = \frac{5}{2} = 2\frac{1}{2}$

3. (C)
   Number of cubes removed = $3 \times 3 - 2 = 7$. Note that the cube at the centre is counted thrice. Number of cubes left = $3 \times 3 \times 3 - 7 = 20$

4. (D)
   Grey ring is linked to white ring and white ring is linked to black ring. Note that grey ring is **NOT** linked to black ring.

5. (D)
   Other diagram can be in the order starting from



6. (D)
Each friend was given 4 cupcakes. Together they ate $5 \times 4 = 20$ cupcakes. At the start, they altogether had $2 \times 20 = 40$ cupcakes.

7. (A)
Lotar finished before Manfred.
Manfred finished before Jan.
Jan finished before Victor.
Also, Eddy finished before Victor.
However, Victor must finish last of these five runners.

8. (B)
Digit 0 is used at 10, 20, 30, 40, 50.
Digit 8 is used at 8, 18, 28, 38, 48, 58.
Hence the final page is 58.

9. (D)
Let the area of the smallest square be 1. Area colored grey = 7 + 9 = 16 and area of the largest square = $4 \times 9 = 36$. Therefore, fraction is $\frac{16}{36} = \frac{4}{9}$.

10. (A)
Let x be the number of apples.
$\frac{x}{6} + 2 = \frac{x}{5}$
$5x + 60 = 6x$
$x = 60$

11. (A)
Assume the covered digits are all zeros.
The sum of the three number is 1243 + 2107 + 0026 = 3376
$\therefore$ 10126 – 3376 = 6750
The covered digits are at tens, hundreds and thousands.
Hence, the covered digits are 5, 6 and 7.

12. (B)
$PQ = QS \quad \rightarrow \Delta PQS$ is isoceles
$\rightarrow \angle PQS = 180 - 2 \times 20 = 140^0$
$PQ = PR \quad \rightarrow \Delta\, PQR$ is isoceles
$\rightarrow \angle PQR = \dfrac{180 - 20}{2} = 80^0$
$\angle RQS = \angle PQS - \angle PQR = 140^0 - 80^0 = 60^0$.

13. (E)

(A) (B) (C) (D)

Figure (E) cannot be formed.

14. (B)

Dora must know other four people. Alan only knows Dora. One such case would be:

– Bella knows Claire and Dora,

– Claire knows Bella, Dora and Erik.

Hence Erik must know Claire and Dora. Therefore, there will be 2 handshakes.

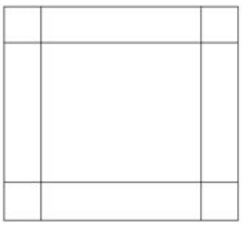
15. (C)

Out of the first 20 shots, she scored $\frac{55}{100} \times 20 = 11$ times.

Out of the total 25 shots, she scored $\frac{56}{100} \times 25 = 14$ times.

Hence, out of the last 5 shots, she scored $14 - 11 = 3$ times.

16. (C)



This is how the square sheet would be cut → 5 pieces that are squares.

17. (D)

$\frac{1}{8}$ of them are dogs.

$1 - \frac{3}{4} = \frac{1}{4}$ of them are cows.

$1 - \frac{2}{3} = \frac{1}{3}$ of them are cats.

Hence, $1 - \frac{1}{8} - \frac{1}{4} - \frac{1}{3} = \frac{(24 - 3 - 6 - 8)}{24} = \frac{7}{24} \times 24 = 7$ of them are Kangaroos.

18. (B)

Length of the rectangle = $\frac{10}{5}$ = 2 cm

Width of the rectangle = $\frac{6}{4} = \frac{3}{2}$ cm

– area of each rectangle = $2 \times \frac{3}{2} = 3$ cm^2

– area of all rectangles = $14 \times 3 = 42$ cm^2

– area of the triangle = $\frac{1}{2} \times 10 \times 6 = 30$ cm^2

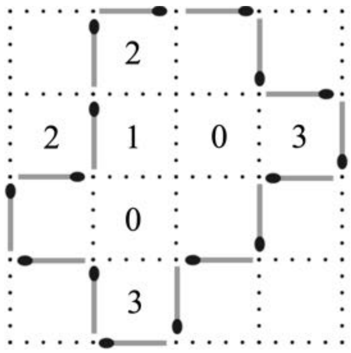
Hence the shaded area = 42 – 30 = 12 cm^2

19. (C)

After 3 hours, first and second candle will last another 3 and 5 hours. Ratio of the bases of the first to second candle is 3:5. Since the ratio of the volume of the first to second candle is 6:8, ratio of the height is

$\frac{6}{3} : \frac{8}{5} = 2 : \frac{8}{5} = 10 : 8 = 5 : 4$

20. (C)



21. (B)

Diameter passing through 7 passes through 23.

– Diameter passing through 1 passes through 17.

– Diameter passing through n passes through 16.

Therefore, $n = 2 \times 16 = 32$.

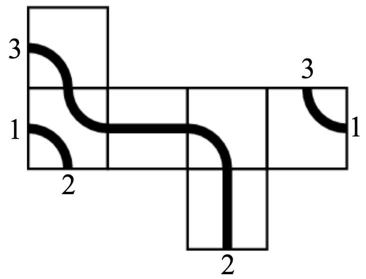
22. (B)

At first, he had 50 Euros. He sells each bottle at $\frac{50+10}{40} = \frac{60}{40} = \frac{3}{2}$ Euros.

He now has $\frac{3}{2} \times 50 = 75$ Euros.

23. (C)
To minimize the number of green sticks used, one green stick is shared by 2 adjacent squares. There are 9 squares in total. Hence, she needs to use at least 5 green sticks. Note: use *Pigeonholes Principle*.

24. (E)



The endpoints numbered the same are the same points when the nets is folded to make a cube.

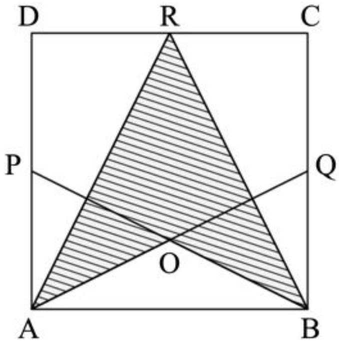
25. (E)
Number of chocolates she has left
$= 60 \times \frac{9}{10} \times \frac{8}{9} \times \frac{7}{8} \times \ldots \times \frac{1}{2} = 6$

26. (A)
5 must be painted differently from both 2 and 6. Similarly, 8 must be painted differently from both 2 and 6. Since there are only three availbale colors, 5 and 8 must have the same color.

27. (C)
Ria : Flora
Before 5 : 3 $(\times 5) \rightarrow$ 25 : 15
After 3 : 5 $(\times 3) \rightarrow$ 9 : 15
Since Flora must have the same amount before and after,
$25 - 9 = 16u = 16$ Euro
$1u = 10$ Euro
Hence at the first, Ria had $25u = 25 \times 10 = 250$ Euro.

28. (E)
Let n be the number of teams.
– Each player will play $3 \times (n - 1) = 3n - 3$ games
– Total number of games $= \frac{3n(3n-3)}{2}$
where $3n$ = total number of players and each game is played by two players.
$\frac{3n(3n-3)}{2} \leq 250 \Longleftrightarrow 9n^2 - 9n - 500 \leq 0$
$\therefore n \leq 7$ where n is a whole number. Hence at most, there are 7 teams.

29. (E)



Let O be the intersection point of AQ and BP

$[\Delta ARB] = \frac{1}{2}[ABCD]$

$[\Delta AOB] = \frac{1}{2}[\Delta ABQ] = \frac{1}{2} \times \frac{1}{4}[ABCD] = \frac{1}{8}[ABCD]$

$$\begin{aligned}[\text{shaded region}] &= [\Delta ARB] - [\Delta AOB] \\ &= \left(\frac{1}{2} - \frac{1}{8}\right)[ABCD] \\ [\text{shaded region}] &= \frac{3}{8}[ABCD]\end{aligned}$$

30. (D)

Assume there is an extra carriage at each end of the train. There are 20 carriages in total. There are $4 \times 199 = 796$ total passengers. Since any block of 5 adjacent carriages carry same number of passengers, the two extra carriages carry same number of passengers as the middle two $= 796 - 700 = 96$ passengers.

KANGAROO MATH THAILAND

Junior

JUNIOR SOLUTION

KANGAROO MATH THAILAND

2019

JUNIOR SOLUTION

1. (D)
   $20 \times 19 + 20 + 19 = 380 + 20 + 19 = 419$

2. (B)
   1 minute 11 seconds $= 71$ seconds
   6 rounds $\rightarrow 6 \times 71$ $= 426$ seconds
   $= 7 \times 60 + 6$
   $= 7$ minutes 6 seconds

3. (E)
   The word should be laterally inverted for every single letter and the order of the letters.

4. (C)
   The smallest sum is $1 + 1 + 1 = 3$
   The largest sum is $6 + 6 + 6 = 18$
   The number of sums from 3 to 18 inclusively is $18 - 3 + 1 = 16$

5. (B)
   Glass (A) contains 6
   Glass (B) contains $6 - \frac{1}{2} \times 5 = 6\frac{1}{2}$

Glass (C) contains $8 - \frac{1}{2} \times 4 = 6$

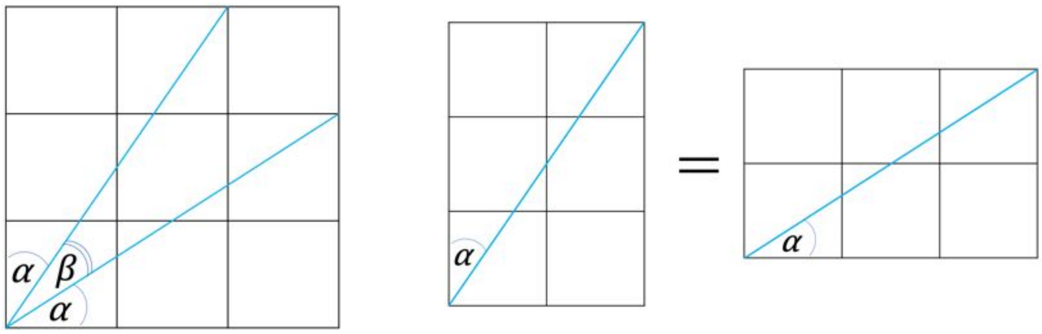
Glass (D) contains $10 - \frac{1}{2} \times 8 = 6$

Glass (E) contains $7 - \frac{1}{2} \times 2 = 6$

6. (B)
5 choices of gates to enter.
There are 5 – 1 = 4 choices of gates to exit (must exit through a different gate that she entered). Therefore there are 5 × 4 = 20 ways.

7. (C)
For the lightest Kangaroo to weigh as heavy as possible.
∴ Their weight difference must be minimum.
∴ Their weights are 31, 32 and 34.

8. (B)

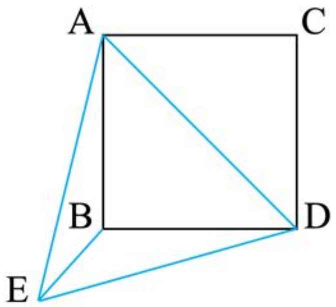


See figure above. We can conclude that $2\alpha + \beta = 90^0$

9. (A)
Option (B), (C), (D) and (E), the total shaded area equal to half the area of the square.
Option (A), the total shaded area is greater than half the area of the square.

10. (B)
Assume the covered digits are all zeros.
The sum of the three number is 15728 + 22004 + 00331 = 38063
Therefore, 57263 – 38063 = 19200
The covered digits are at hundreds, thousands and ten thousands. Hence the covered digits are 1, 2 and 9.

11. (C)



$\angle CBE = \angle ABE$

$$\angle CBE = \frac{360^0 - \angle ABC}{2}$$

$$= \frac{360^0 - 90^0}{2} = 135^0$$

12. (C)

For $\frac{a}{b} + \frac{c}{d}$ to be the least, b and d must be the largest and a and c must be the least. Therefore

$$\frac{a}{b} + \frac{c}{d} = \frac{1}{9} + \frac{2}{10} = \frac{5+9}{45} = \frac{14}{45}$$

13. (E)

Let a and b be the width and length of the white rectangle. The length of black rectangle = $3a$ and the width of black rectangle = $\frac{b}{3}$ (equal area).

Therefore

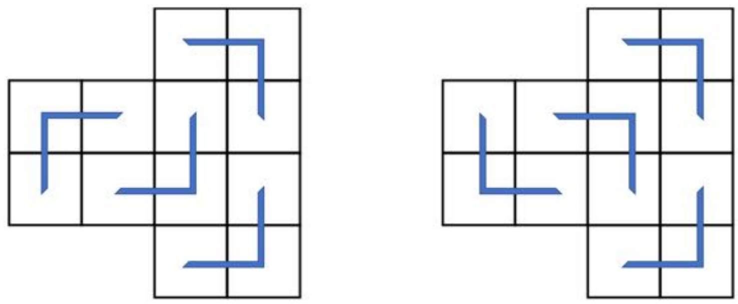
$$\frac{3a}{\frac{b}{3} + b} = \frac{3}{5} \rightarrow \frac{3a}{\frac{4b}{3}} = \frac{3}{5}$$

$$\frac{a}{b} \times \frac{9}{4} = \frac{3}{5}$$

Hence

$$\frac{a}{b} = \frac{12}{45} = \frac{4}{5}.$$

14. (B)



15. (D)

Swimming $= 1 - \frac{3}{4} - \frac{1}{5} = \frac{20 - 15 - 4}{20} - \frac{1}{20}$

Hence total distance = 20 × 2 = 40 km.

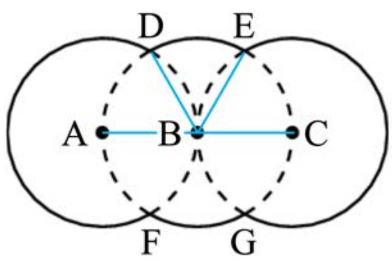
16. (B)

$2l$ of diluted juice requires $\frac{1}{8} \times 2 = \frac{1}{4}$ l of concentrate. Amount of juice concentrate available is $\frac{1}{2}$ l. Hence fraction to be used is

$$\frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

17. (A)

∠ DBA = ∠ DBE = ∠ EBC = 600



Perimeter of the shape = major arc DF + arc DE + arc FG + major arc EG

$$= \frac{2}{3} \times 2\pi R + \frac{1}{6} \times 2\pi R + \frac{1}{6} \times 2\pi R + \frac{2}{3} \times 2\pi R$$

$$= \frac{5}{3} \times 2\pi R$$

$$= \frac{10\pi R}{3}$$

18. (C)

The sum of the digits is $3a + 4b$ and $\overline{ab} = 10a + b$.

Therefore,

$3a + 4b = 10a + b$

$7a = 3b$

$\therefore a = 3, b = 7$

Hence $a + b = 3 + 7 = 10$.

19. (D)

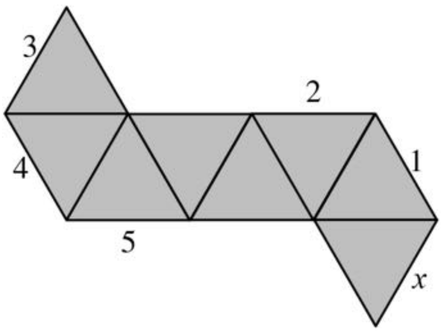
To maximise the number of boxes, we pack number of pears in each box in the manner of 1, 2, 3 ...

Therefore, $60 = 1 + 2 + ... + 10 + 5$

The extra 5 pears can be packed with the box with 10 pears thus 15 pears.

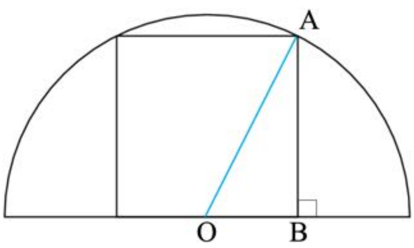
For example 1, 2, 3, ..., 8, 9, 15 → 10 boxes. Also, 60 apples are divisible by 10 boxes. Hence the largest possible number of boxes that can be packed in this way is 10.

20. (E)



Segmen 1 and 4 will coincide. Similarly, for 2 and 3 and also x and 5.

21. (A)



$OA = 1$

$OB = \dfrac{AB}{2}$

Using Pythagoras' Theorem:

$OA^2 = OB^2 + AB^2 \rightarrow 1 = \dfrac{AB^2}{4} + AB^2$

$1 = \dfrac{5}{4}AB^2 \rightarrow AB^2 = \dfrac{4}{5}$

Hence the area of the square $= AB^2 = \dfrac{4}{5}$ cm^2

22. (E)

Let O, A and B be the centre, closer dot and further dot respectively.

O A 3 B

$AB = 3$

Perimeter of circle with radius $OA = 2\pi OA$.

Perimeter of circle with radius $OB = 2\pi(OA + 3)$.

Therefore,

$$\frac{2\pi(OA+3)}{2\pi OA} = 2.5$$

$$OA + 3 = 2.5OA$$

$$1.5OA = 3$$

$$OA = 2$$

Hence the distance from the centre to the far point is $OB = OA + AB = 2 + 3 = 5$ cm.

23 (B)

Only way to have 444 as a triplet , 17 must be part of 44, 45.

The number of digits before 444

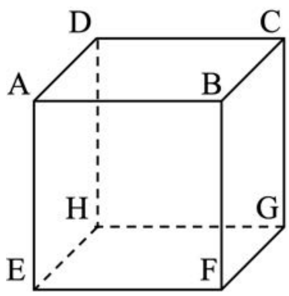
$= 1 + 2 + 3 + \ldots + 43$

$= \frac{43 \times 44}{2}$ which is not divisible by 3

Conclusion: **Contradiction**

24 (D)

For the plane to pass through exactly three vertices, it must pass through a diagonal on a face and a vertex on the opposite face (but not on the corresponding diagonal), see figure.

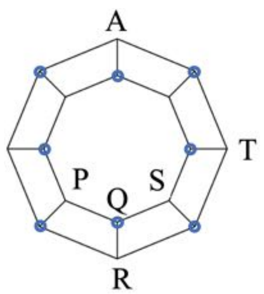


For example A, C and F or A, C and H.

B, D and E or B, D and G.

Similarly if we use diagonal EG or FH from the bottom face. Hence in total 8 such planes.

25 (C)



2019 is an odd number. After an odd number of moves, it can only land on the circled vertices.

26 (C)

b and c must be odd.

Since $c = 2b + 1$

$\rightarrow$ c must start and end with digit 7

$\rightarrow$ b must start and end with digit 3

$\rightarrow$ a must start and end with digit 1

Try $a = 151 \rightarrow b = 303$ but c is not correct.

$a = 161 \rightarrow$ similar

$a = 171 \rightarrow$ similar

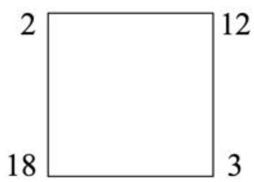
$a = 181 \rightarrow b = 363 \rightarrow c = 727$

$a = 191 \rightarrow b = 383 \rightarrow c = 767$

Hence 2 possible values for a.

27 (D)

1 cannot be used as it is a factor of any integer. Start from 2 at one vertex and 3 from its diagonal. Other two numbers must be both multiple of 6 (because each needs to be divisible by 2 and 3).



Therefore, the two numbers are 12 and 18.

Hence the sum is 2 + 3 + 12 + 18 = 35.

28 (B)

Must delete 2 i.e. 10 and 70. The product of the remaining elements 20×80 is perfect square. Therefore;

$$
\begin{aligned}
30 \times 40 \times 50 \times 60 \times 90 &= 3 \times 4 \times 5 \times 6 \times 9 \times 10^5 \\
&= 3^4 \times 2^2 \times 5 \times 2 \times 10^5 \\
&= 3^4 \times 2^2 \times 10^6 \text{ is perfect square.}
\end{aligned}
$$

29 (A)

$$[\Delta APQ] = 6[\Delta ABD] = 6 \times \frac{1}{2} \times s = 3s$$

$$[\Delta AQR] = 12[\Delta ACD] = 12 \times \frac{1}{2} \times s = 6s$$

$$[\Delta APR] = 8[\Delta ABC] = 8s$$

$$[\Delta PQR] = [\Delta APQ] + [\Delta AQR] - [\Delta APR]$$

$$[\Delta PQR] = 3s + 6s - 8s$$

$$[\Delta PQR] = s$$

30 (C)

Let the 4–digit number be $\overline{abcd},\ a \geq 1$.

$\therefore \overline{abc}\,|\,\overline{abcd} \rightarrow$ deleting d

$\therefore \overline{abc}\,|\,\overline{d} \rightarrow d = 0$

Similarly,

$\therefore \overline{abo}\,|\,\overline{abco} \rightarrow$ deleting c

$\therefore \overline{ab}\,|\,\overline{abc}$

$\therefore \overline{ab}\,|\,\overline{c} \rightarrow c = 0$

deleting a, $\overline{boo}\,|\,\overline{aboo} \rightarrow b\,|\,\overline{ab}$

deleting b, $\overline{aoo}\,|\,\overline{aboo} \rightarrow a\,|\,\overline{ab}$

$\therefore \overline{ab}$ must be divisible by a and b

$\therefore \overline{ab}$ = 11, 22, ..., 99 ($a = b$)

or $\overline{ab}$ = 12, 15, 24, 36, 48 ($a \neq b$)

Hence in total there are 9 + 5 = 14 such numbers.

KANGAROO MATH THAILAND

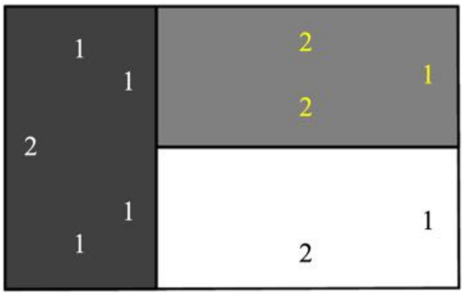
Student

STUDENT SOLUTION

KANGAROO MATH THAILAND
2019

STUDENT SOLUTION

1. (A)



2. (E)

a	b
c	d

Sum of the numbers in row; $a + b$ and $c + d$. Sum of the numbers in column; $a + c$ and $b + d$. Therefore, sum of these:
$2(a + b + c + d) = 2(1 + 2 + 3 + 4) = 20$

Two of these sums are 4 and 5. Therefore, sum of other two sums is $20 – (4 + 5) = 11$.

3. (E)
Option (A), (B) and (C) → the shaded part has the same area as the non-shaded part.

Option (D) → the shaded part has smaller area than the non-shaded part.
Option (E) → the shaded part has larger area than the non-shaded part.

4. (D)
Grey triangle is linked to white triangle and white triangle is linked to black triangle. Note that grey triangle is NOT linked to black triangle.

5. (C)
Ignoring the edges at the base, each triangle face has two edges. However, each edge is shared by two triangle faces. There are

$\frac{23 \times 2}{2} = 23$ edges, ignoring the edges at the base.

It means 23 triangular faces, therefore 23 edges at the base. Hence in total there are 23 + 23 = 46 edges.

6. (B)
Assume the covered digits are all zeros.
∴ The sum of the three numbers is
7243 + 2107 + 0026 = 9376
∴ 11126 – 9376 = 1750

The covered digits are at tens, hundreds and thousands.
Hence the covered digits are 1, 5 and 7.

7. (B)
To minimise the positive integer, all digits, possibly except the leftmost digit, are 9s.
$2019 = 9 \times 224 + 3$
Therefore, the number is

$$3\underbrace{99---9}_{224 \text{ of } 9\text{s}}$$

Hence the leftmost digit is 3.

8. (C)
The probability of rolling a 1, 2 and 3 is $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$, respectively.
Therefore, the number of faces marked with 1, 2 and 3 dots is 3, 2 and 1, respectively.
Hence option (C) is impossible as there are two faces marked with 3 dots.

9. (D)
$(a \diamond b) \diamond c = c - (a \diamond b) = c - (b - a) = a - b + c$
$a \diamond (b \diamond c) = (b \diamond c) - a = (c - b) - a = -a - b + c$

Therefore,

$a - b + c = - a - b + c$

$2a = 0 \rightarrow a = 0$

10. (D)

Multiples of 2^{10} from 2^{10} to 2^{13} inclusively are

$1 \times 2^{10} = 2^{10},\ 2 \times 2^{10},\ 3 \times 2^{10},\ \ldots, 8 \times 2^{10} = 2^3 \times 2^{10} = 2^{13}$

Hence there are 8 multiples of 2^{10} from 2^{10} to 2^{13} inclusively.

11. (D)

$7! + 8! + 9! = 7! \times (1 + 8 + 72) = 7! \times 81$

Highest power of 3 dividing 7! is 3^2 (from 3 and 6).

Highest power of 3 dividing 81 is 3^4.

Hence highest power of 3 dividing their product is 3^6.

12. (B)

	Boys	Girls
Before	$10a$	$10b$
After	$12a$	$8b$

$\therefore 10a + 10b + 1 = 12a + 8b$

$2a - 2b = 1$

$\therefore$ the number of students now $= 12a + 8b$

$= 6(2a - 2b) + 20b$

$= 6 + 20b$

When $b = 1 \rightarrow 6 + 20b = 26$

13. (E)

Let a, b and c be the length, width and height of the container (in m).

Area of the three bases:

$\frac{120}{2} = 60\ \text{m}^2,\ \frac{120}{3} = 40\ \text{m}^2,\ \frac{120}{5} = 24\ \text{m}^2$

Say $ab = 60$, $bc = 40$ and $ca = 24$, therefore, $(abc)^2 = 60 \times 40 \times 24 = 57600 = 240^2$.

Hence the volume of the container is $abc = 240\ \text{m}^3$.

14. (E)

Carl is not wearing a hat $\rightarrow$ Bob wears a hat.

However, we cannot determine if Alex wears a hat or not.

Note: concept used is $p \Rightarrow q \equiv \overline{q} \Rightarrow \overline{p}$.

15. (D)

Without the long pulley, the middle pulley will be up by $\frac{24}{2} = 12$ cm.

With the long pulley, point Q will be up by $\frac{12}{2} = 6$ cm.

16. (C)

The three good positive integers are 12, 9 and 8. Possible largest divisor of n (excluding n):

$\frac{n}{2} = n - 6 \rightarrow n = 12$

$\frac{n}{3} = n - 6 \rightarrow n = 9$

$\frac{n}{4} = n - 6 \rightarrow n = 8$

17. (A)

Mary wins if she draws the fruit chew on the 2nd or 4th draw and John always draws chocolates.

P_r (Mary wins) = P_r (Mary wins at 2nd draw) + P_r (Mary wins at 4nd draw)

$$P_r \text{ (Mary wins)} = \frac{4}{5} \times \frac{1}{4} + \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} \times \frac{1}{2}$$
$$= \frac{1}{5} + \frac{1}{5} = \frac{2}{5}$$

18. (B)

Area of the shaded triangle = area of the whole figure – (sum of 3 unshaded triangles)

$$= a^2 + b^2 - \{\frac{1}{2}a \times a + \frac{1}{2}b(b - a) + \frac{1}{2}b(a + b)\}$$
$$= a^2 + b^2 - \{\frac{1}{2}a^2 + \frac{1}{2}b^2 - \frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}b^2\}$$
$$= \frac{1}{2}a^2$$

19. (A)

$\sqrt{20} < 5$

$20 + \sqrt{20} < 25$

$\sqrt{20 + \sqrt{20}} < \sqrt{25} = 5$

Repeat the process, we get:

$$\sqrt{20 + \sqrt{20 + \sqrt{20 + \sqrt{20 + \sqrt{20}}}}} < 5$$

Since

$$\sqrt{20+\sqrt{20+\sqrt{20+\sqrt{20+\sqrt{20}}}}} > \sqrt{20} > 4$$

Hence the integer part of the number is 4.

20. (E)

$a + \dfrac{b}{c} = 11$ (i)

$b + \dfrac{a}{c} = 14$ (ii)

From (i) and (ii) we get $a + b + \dfrac{a+b}{c} = 25$. Both a and b are divisible by c.

Therefore, $a + b$ is divisible by c.

$a + b + \dfrac{a+b}{c} = 25 \rightarrow \dfrac{a+b}{c}(c+1)$

$\dfrac{a+b}{c}$ is a factor of 25.

If $\dfrac{a+b}{c} = 1$ than $a + b = c$ is not possible since both a and b need to be divisible by c.

Hence $\dfrac{a+b}{c} = 5$.

21. (B)

$1024 = 2^{10}$

$a = 1 + 2^1 + 2^2 + \ldots + 2^{10} = 2^{11} - 1$

$b = 1 \times 2^1 \times 2^2 \times \ldots \times 2^{10} = 2^{55}$

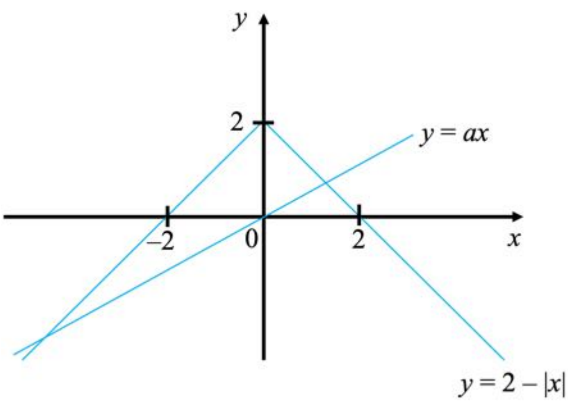
Therefore

$(a + 1) = 2^{11}$

$(a + 1)^5 = 2^{55} = b$

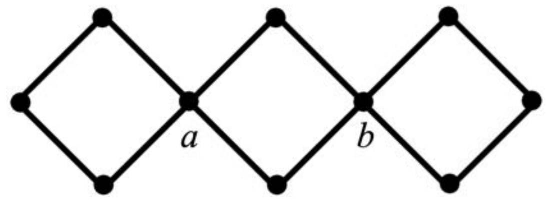
Hence $(a + 1)^5 = b$

22. (B)



Line representing $y = ax$ must intersect the origin and has a gradient of a.
For $2 - |x| = ax$ to have two solutions, the two graphs must intersect each other at two distinct points $\rightarrow a \in (-1, 1)$

23. (C)



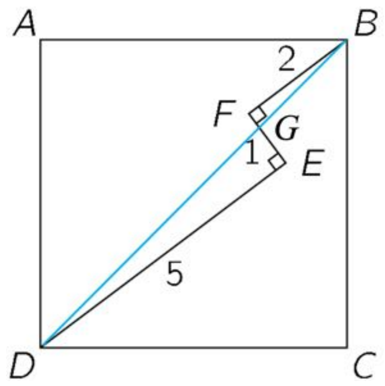
Let numbers in the middle be a and b (as shown in the diagram). Sum of the numbers on the three squares:
$3S = (1 + 2 + ... + 10) + a + b$
$3S = 55 + a + b \geq 55 + 1 + 2$
$S \geq \frac{58}{3} = 19\frac{1}{3} \rightarrow$ least of S is 20.

24. (E)
There are three kinds of such planes:
(1) Cube faces = 6
(2) Passing through 4 points (opposite edges) = 6
(3) Passing through only 3 vertices = 8
Hence in total = 6 + 6 + 8 = 20.

25. (A)
Suppose $y = ax$ is one of the four lines.
$\therefore\ ax = x^2 - 2 \Leftrightarrow x^2 - ax - 2 = 0$
The product of the two roots is –2.
$\therefore$ The product of the x–coordinates of the two points is –2. Hence the product of the x–coordinates of the eight points is $(-2)^4 = 16$.

26. (D)
$|n^2 - 2n - 3| = |n + 1| \times |n - 3|$
For it to be a prime number,
$|n + 1| = 1 \rightarrow n = 0$ or -2
$|n - 3| = 1 \rightarrow n = 4$ or 2
Hence there are 4 possible integers n.

27. (E)



Let BD intersect EF at point $G \rightarrow \Delta BFG \sim \Delta DEG$.

$\dfrac{FG}{EG} = \dfrac{BF}{DE} = \dfrac{2}{5}$

Since $FG + EG = EF = 1$

$\therefore\ EG = \dfrac{5}{7},\ FG = \dfrac{2}{7}$

Using Pythagoras' Theorem:

$BG = \sqrt{4 + \dfrac{4}{49}} = \dfrac{10}{7}\sqrt{2}$

$DG = \sqrt{25 + \dfrac{25}{49}} = \dfrac{25}{7}\sqrt{2}$

Hence $BD = 5\sqrt{2}$ and $AB = 5$.

28. (C)

a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
49	196	289	400	25	64	121	25

Except the first 4 terms, the pattern repeats with every cycle of 3 terms, 25, 64 and 121.

$2019 - 4 = 2015$

$2015 = 3 \times 671 + 2$

Hence $a_{2019} = 64$

29. (B)

There are $C_3^{10} = 120$ ways to choose 3 numbers from 10. The desired events are:

(1, 2, 3), (1, 3, 5), (1, 4, 7), (1, 5, 9)

(2, 3, 4), (2, 4, 6), (2, 5, 8), (2, 6, 10)

(3, 4, 5), (3, 5, 7), (3, 6, 9)

(4, 5, 6), (4, 6, 8), (4, 7, 10)

(5, 6, 7), (5, 7, 9)
(6, 7, 8), (6, 8, 10)
(7, 8, 9)
(8, 9, 10)
There are 20 ways. Hence the required probability is $\dfrac{20}{120} = \dfrac{1}{6}$.

30. (C)

1	2	4	5	3
3	1	2	4	5
5	3	1	2	4
4	5	3	1	2
2	4	5	3	1