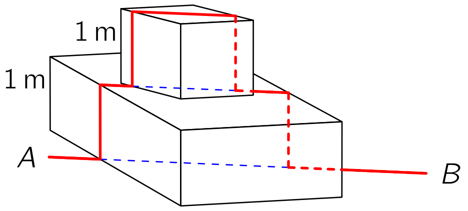
3 points

 # 1. What is the sum of the last two digits of the product $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$?

(**A**) 2 (**B**) 4 (**C**) 6 (**D**) 8 (**E**) 16

SOLUTION:

 # 2. An ant walked every day on a straight horizontal line path from A to B, which are 5 m apart. One day humans placed on its path two strange obstacles of height 1 m each. Now the ant walks along or above the same straight line except that it now has to climb up and down vertically over both the two obstacles, as in the picture. How long is its path now?

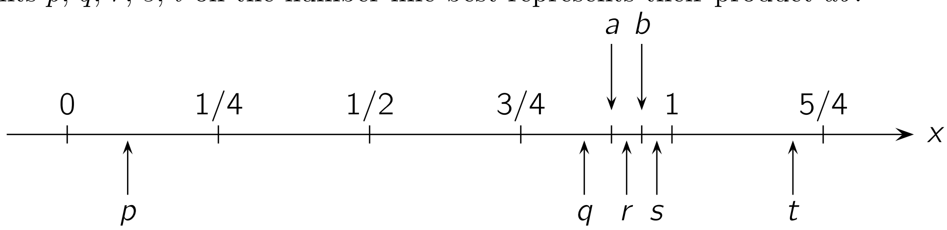


(**A**) 7 m (**B**) 9 m (**C**) $5 + 4\sqrt{2}$ m (**D**) $9 - 2\sqrt{2}$ m

(**E**) the length depends on the angles the obstacles are situated along the path

SOLUTION: The horizontal part of the path is exactly as long as the original path AB. The can be seen by projecting the new path onto the horizontal plane. The extra length comes from the four vertical parts of the path. The total length is 5+(1+1+1+1)=9 m.

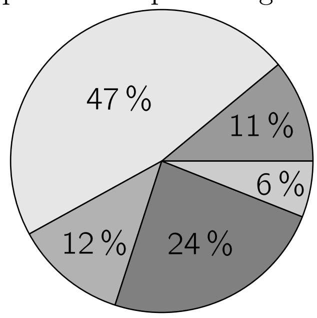
 # 3. Rene marked two points a and b as accurately as possible on the number line. Which of the points p, q, r, s, t on the number line best represents their product ab?



(**A**) p (**B**) q (**C**) r (**D**) s (**E**) t

SOLUTION: Both a and b are less than 1 so their product is less than 1. In fact $ab < a$ which excludes points r, s, t. Also certainly $a, b > \frac{1}{2}$ so $ab > \frac{1}{4}$so certainly p is excluded. This leaves q which, moreover, is realistically placed because $b \approx 1$ so $ab \approx a$, so ab is close but less that a. Such is the case as with the location of q.

 # 4. The pie chart shows how the students of my school get to school. Approximately twice as many go by bike as use public transport and roughly the same number come by car as walk. The rest use a moped. What percentage use a moped?



(**A**) 6 % (**B**) 11 % (**C**) 12 % (**D**) 24 % (**E**) 47 %

SOLUTION:

 # 5. The sum of five three-digit numbers is 2664, as shown on the board. What is the value of $A+B+C+D+E$?

$$\begin{array}{r} \mathsf{A\,B\,C} \\ +\,\mathsf{B\,C\,D} \\ +\,\mathsf{C\,D\,E} \\ +\,\mathsf{D\,E\,A} \\ +\,\mathsf{E\,A\,B} \\ \hline \mathsf{2\,6\,6\,4} \end{array}$$

(**A**) 4 (**B**) 14 (**C**) 24 (**D**) 34 (**E**) 44

SOLUTION: $(100A+10B+C)+(100B+10C+D)+(100C+10D+E)+(100D+10E+A)+(100E+10A+B) = 111(A+B+C+D) = 2664$ so $A+B+C+D = 2664 : 111 = 24$.

 # 6. What is the value of $\dfrac{1010^2+2020^2+3030^2}{2020}$?

(**A**) 2020 (**B**) 3030 (**C**) 4040 (**D**) 6060 (**E**) 7070

SOLUTION: $\dfrac{1010^2+2^2\cdot 1010^2+3^2\cdot 1010^2}{2\cdot 1010} = \dfrac{(1+2^2+3^2)1010^2}{2\cdot 1010}$.

 # 7. Let a, b and c be integers satisfying $1 \leqslant a \leqslant b \leqslant c$ and $abc = 1\,000\,000$. What is the largest possible value of b?

(**A**) 100 (**B**) 250 (**C**) 500 (**D**) 1000 (**E**) 2000

SOLUTION:

 # 8. If D dogs weigh K kilograms and E elephants weigh the same as M dogs, how many kilograms does one elephant weigh?

(**A**) $DKEM$ (**B**) $\frac{DK}{EM}$ (**C**) $\frac{KE}{DM}$ (**D**) $\frac{KM}{DE}$
(**E**) $\frac{DM}{KE}$

SOLUTION:

 # 9. There are two dice. Each one has two red faces, two blue faces and two white faces. If we roll both dice together, what is the probability that both show the same color?

(**A**) $\frac{1}{12}$ (**B**) $\frac{1}{9}$ (**C**) $\frac{1}{6}$ (**D**) $\frac{2}{9}$ (**E**) $\frac{1}{3}$

SOLUTION:

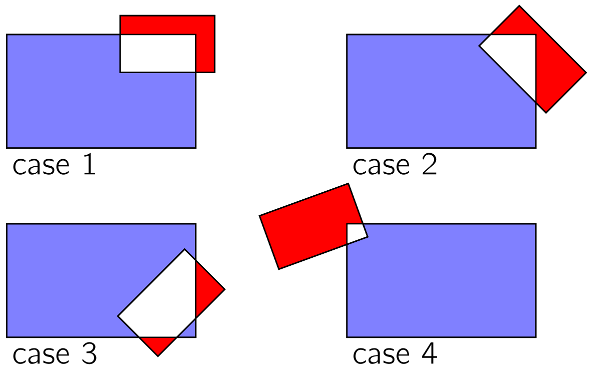
 # 10. Which of the following numbers is not divisible by 3 for any integer n?

(**A**) $5n+1$ (**B**) n^2 (**C**) $n(n+1)$ (**D**) $6n-1$ (**E**) n^3-2

SOLUTION:

4 points

 # 11. A blue rectangle and a red rectangle are overlapping. The figure shows 4 different such cases. We denote by B the area of the part of the blue rectangle that is not common to the two rectangles, and we denote by R the area of the red rectangle that is not common to the two. Which of the following statements is true about the quantity $B-R$?



(**A**) In case 1 the quantity B-R is larger than in the other cases

(**B**) In case 2 the quantity B-R is larger than in the other cases

(**C**) In case 3 the quantity B-R is larger than in the other cases

(**D**) In case 4 the quantity B-R is larger than in the other cases

(**E**) The quantity B-R is the same in all cases

Solution: If W denotes the common area then in each case B-R= (B+W)-(R+W)= (area of blue rectangle)-(area of red rectangle) = constant.

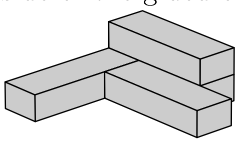
 # 12. Five coins are lying on a table with the "heads" side up. At each step you must turn over exactly three of the coins. What is the least number of steps required to have all the coins lying with the "tails" side up?

(**A**) 2 (**B**) 3 (**C**) 4 (**D**) 5

(**E**) It's not possible to have all the coins with their "tails" side up.

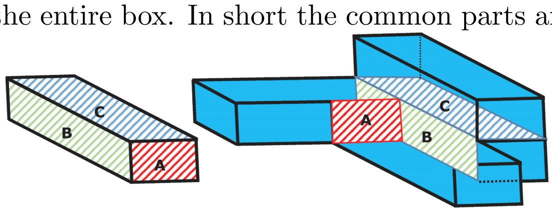
Solution:

 # 13. Four identical boxes are glued together to make the shape shown in the picture. One litre of paint is needed to paint the outside of one such box. How many litres of paint are needed to paint the outside of the glued construction?



(**A**) 2.5 (**B**) 3 (**C**) 3.25 (**D**) 3.5 (**E**) 4

Solution: We would need 4 litres of paint but because of overlaps (where two boxes meet), we need to subtract the areas that are covered. They are of three types. a) The two red areas A, b) the two green areas B and c) the two blue areas C. However, observing the box on the left, two A's, two B's and two C's make the entire box. In short the common parts amount to an entire box, so we need 4-1



= 3 litres of paint.

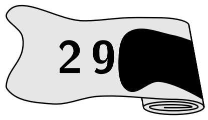
 # 14. Let a, b and c be integers. Which of the following is certainly NOT equal to $(a-b)^2+(b-c)^2+(c-a)^2$?

(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 6 (**E**) 8

Solution: One is not possible since if one of the squares is non-zero, at least one another is as well. Others are possible:

$$\begin{cases} (0-0)^2+(0-0)^2+(0-0)^2=0+0+0=0, \\ (1-0)^2+(0-1)^2+(1-1)^2=1+1+0=2, \\ (1-2)^2+(2-3)^2+(3-1)^2=1+1+4=6, \\ (2-0)^2+(0-2)^2+(2-2)^2=4+4+0=8. \end{cases} \quad \text{and}$$

 # 15. The first two digits of a 100-digit integer are 2 and 9. How many digits does the square of this number have?

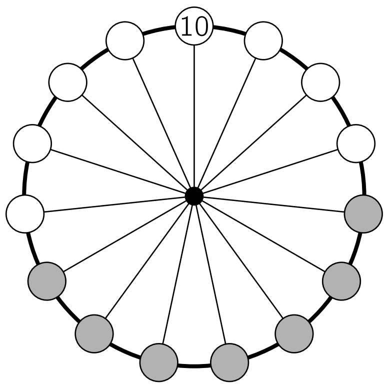


(**A**) 101 (**B**) 199 (**C**) 200 (**D**) 201

(**E**) It cannot be determined

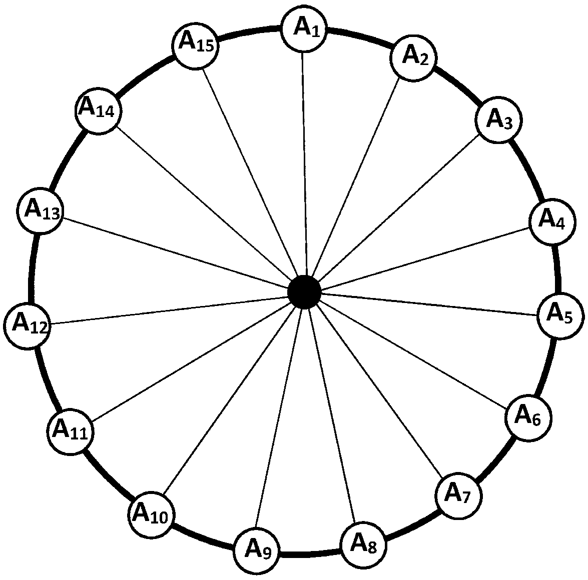
Solution: $10^{99} < A < 30 \times 10^{98}$, so $10^{198} < A^2 < 900 \times 10^{196} < 1000 \times 10^{196} = 10^{199}$. It follows that A^2 has 199 digits.

 # 16. Matjaz has placed 15 numbers on a wheel. Only one of the numbers is visible, the 10 at the top. The sum of the numbers in any 7 consecutive positions on the wheel, such as the ones shaded grey, is always the same. When all 15 numbers are added, exactly how many of the numbers 75, 216, 365 and 2020 are possible totals?

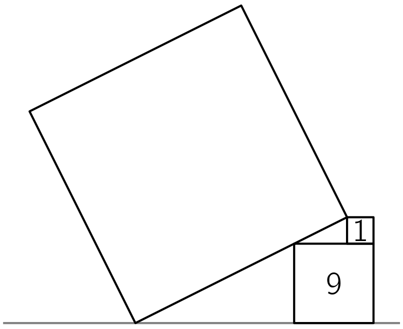


(**A**) zero (**B**) one (**C**) two (**D**) three (**E**) four

Solution: Call the numbers A_n $n = 1$ to 15, with $A_1 = 10$. Since $A_1 + \ldots + A_7 = A_2 + \ldots + A_8$, it follows that $A_1 = A_8$. Similarly $A_8 = A_{15}$, so in fact $A_1 = A_8 = A_{15}$. If we continue around the wheel we will find that $A_1 = A_8 = A_{15} = A_7 = A_{14} = \ldots$ etc. As a matter of fact in this list of equal numbers you will find all fifteen of $A_1, \ldots, A_{15}$. This is fairly obvious since A_{15} is the immediate predecessor of A_1, so the next equal number (here A_7) is the immediate predecessor of a number in the list. So few turns around the wheel in jumps of 7 will pass from all the numbers on the wheel. In other words, all the numbers in the list are equal to each other, namely equal to 10. So the sum of all the numbers in the list is $15 \times 10 = 150$, no other possibility arising. So the answer is "none".



 # 17. A large square touches two other squares, as shown in the diagram. The numbers in the small squares represent their areas. What is the area of the large square?



(**A**) 49 (**B**) 80 (**C**) 81 (**D**) 82 (**E**) 100

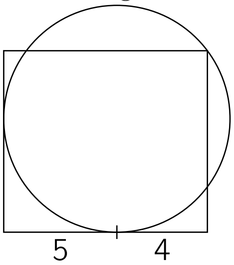
SOLUTION:

 # 18. The sequence f_n is given by $f_1 = 1$, $f_2 = 3$ and $f_{n+2} = f_n + f_{n+1}$ for $n \geq 1$. How many of the first 2020 elements of the sequence are even?

(**A**) 673 (**B**) 674 (**C**) 1010 (**D**) 1011 (**E**) 1347

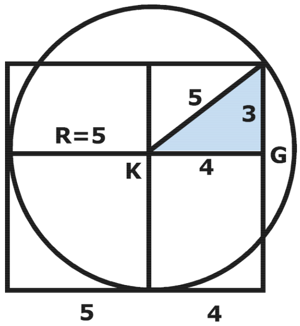
SOLUTION: We look at parity. The first few terms are $1, 1, 0, 1, 1, 0, 1, 1, ...$ Note that the forth and fifth terms are a repetition of the first two and it is clear that the sequence is periodic. Namely, it has the three terms $1, 1, 0$ repeating eternally, with a zero (even number) every third term. As $2020 = 673 \times 3 + 1$ we have in total 673 zero's.

 # 19. A circle and a rectangle have been drawn in such a way that the circle touches two of the sides of the rectangle and passes through one of its vertices. The distances of two vertices of the rectangle from one of the points where the circle touches the rectangle are 5 and 4, as shown. What is the area of the rectangle?

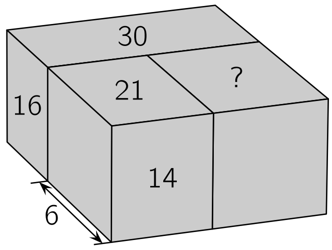


(**A**) 27π (**B**) 25π (**C**) 72 (**D**) 63
(**E**) none of the previous

SOLUTION: Drawing the perpendiculars to the sides of rectangle at the touching points E and F, we note that they pass from the centre K of the circle. It follows that the radius R = KF = AE = 5. So from Pythagoras on KGC we get GC=3 and so the vertical side of the rectangle has length AD=3+5=8. The required area is $8 \times 9 = 72$.

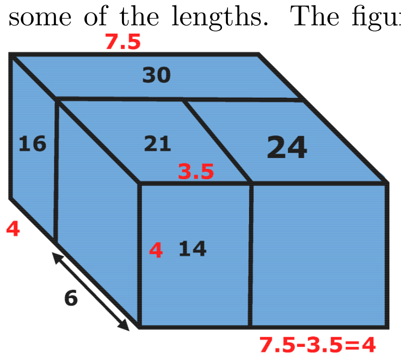


 # 20. Three cuboids are arranged to make a larger cuboid as in the figure. The width of one of them is 6 and the areas of some of their faces are 14, 21, 16, 30, as shown. What is the area of the face with the question mark?



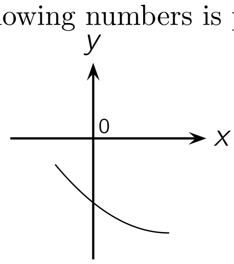
(**A**) 18 (**B**) 24 (**C**) 28 (**D**) 30
(**E**) cannot be determined

SOLUTION: From the face of area 21 and width 6, we conclude that its length is 3.5. Using this it is easy to determine some of the lengths. The figure describes the results. So the required area is $(7.5 - 3.5) \times 6 = 24$.



5 points

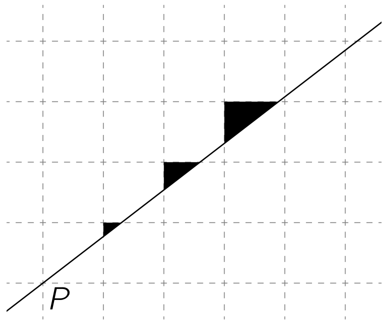
 # 21. The figure shows a section of the parabola with equation $y = ax^2 + bx + c$. Which of the following numbers is positive?



(**A**) c (**B**) $b+c$ (**C**) ac (**<u>D</u>**) bc (**E**) ab

Solution:

 # 22. On a square grid paper, a little kangaroo draws a line passing through the lower left corner P of the grid and colours in three triangles as shown.

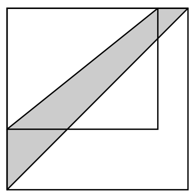


Which of the following could be the ratio of the areas of the triangles?

(**A**) 1 : 2 : 3 (**B**) 1 : 2 : 4 (**C**) 1 : 3 : 9 (**D**) 1 : 4 : 8
(**<u>E</u>**) None of the previous is correct

Solution: Similar triangles with side ratios 1:2:3

 # 23. The length of one of the sides of a rectangular garden is increased by 20% and the length of the other side is increased by 50%. The new garden is a square, as shown in the diagram. The shaded area between the diagonal of the square garden and the diagonal of the original rectangular garden is $30\,\text{m}^2$. What was the area of the original rectangular garden?



(**A**) $60\,\text{m}^2$ (**B**) $65\,\text{m}^2$ (**C**) $70\,\text{m}^2$ (**<u>D</u>**) $75\,\text{m}^2$ (**E**) $80\,\text{m}^2$

Solution: Call $6x$ the side of the square, so then the rectangle has sides $5x$ and $4x$. The shaded area can be divided in two triangles of areas $(x \cdot 4x)/2$ and $(2x \cdot 6x)/2$.

 # 24. A large integer N is divisible by all except two of the integers from 2 to 11. Which of the following pairs of integers could be these exceptions?

(**A**) 2 and 3 (**B**) 4 and 5 (**C**) 6 and 7 (**<u>D</u>**) 7 and 8 (**E**) 10 and 11

Solution:

 # 25. In the morning, the ice-cream shop offers 16 flavours. Anna wants to choose a 2-flavour ice cream. In the evening several flavours are sold out and Bella wants to choose a 3-flavour ice cream from those flavours left. Both Anna and Bella can choose from the same number of possible combinations. How many flavours were sold out?

(**A**) 2 (**B**) 3 (**C**) 4 (**D**) 5 (**<u>E</u>**) 6

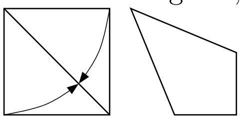
Solution:

 # 26. Tony has 71 marbles at his disposal in a box. He is allowed to take out exactly 30 marbles from the box or to return exactly 18 marbles to it. Tony is allowed to apply each operation as many times as he wishes. What is the smallest number of marbles than can be in the box?

(**A**) 1 (**B**) 3 (**C**) 5 (**D**) 7 (**E**) 11

Solution: $(30, 18) = 6$ and the biggest number less than 71 and divisible by 6 is 66. Thus, $71 - 66 = 5$ is the least possible number of marbles that could remain in the box. One possible realization is $71 - 30 - 30 + 18 + 18 + 18 - 30 - 30 = 5$.

 # 27. Wajda took a square piece of paper of side 1 and folded two of its sides to the diagonal, as shown in the diagram, to make a quadrilateral. What is the area of this quadrilateral?



(**A**) $2 - \sqrt{2}$ (**B**) $\frac{\sqrt{2}}{2}$ (**C**) $\sqrt{2} - 1$ (**D**) $\frac{7}{10}$ (**E**) $\frac{3}{5}$

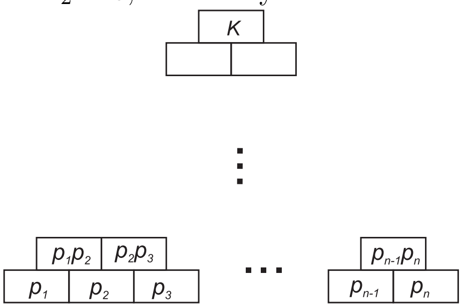
Solution:

 # 28. An iceberg has the shape of a cube. Exactly 90% of its volume is hidden below the surface of the water. Three edges of the cube are partially visible over the water. The visible parts of these edges are 24m, 25m and 27m. How long is an edge of the cube?

(**A**) 30 m (**B**) 33 m (**C**) 34 m (**D**) 35 m (**E**) 39 m

Solution:

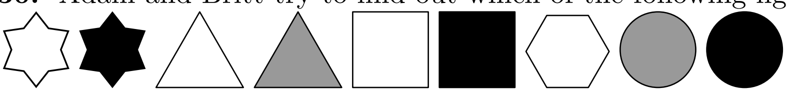
 # 29. There are n different prime numbers p_1 to p_n written from left to right in the bottom row of the table shown. The product of two numbers next to each other in the same row is written in the box directly above them. The number $K = p_1^{\alpha_1} p_2^{\alpha_2} \ldots p_n^{\alpha_n}$ is written in the box in top row. In a table where $\alpha_2 = 8$, how many numbers are divisible by the number p_4?



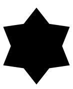
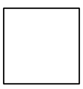
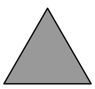
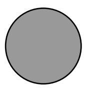
(**A**) 4 (**B**) 16 (**C**) 24 (**D**) 28 (**E**) 36

Solution:

 # 30. Adam and Britt try to find out which of the following figures is Carl's favourite.



Adam knows that Carl has told Britt its shape. Britt knows that Carl has told Adam its colour. Then the following conversation takes place. Adam: "I don't know Carl's favourite figure and I know that Britt doesn't know it either." Britt: "At first I didn't know Carl's favourite figure, but now I do." Adam: "Now I know it too." Which figure is Carl's favourite?

(**A**)  (**B**)  (**C**)  (**D**)  (**E**) 

Solution: