Junior Maths Mastery Challenge Sample

Paper F

Section A

Questions 1 to 5 carry 3 marks each.

1. Find the value of the following.

$$11 + 22 + 33 + \ldots + 528 + 539 + 550$$

[Arithmetic]

11 + 550 = 561
Using the rainbow method, each pair of terms gives a sum of 561.
There are 50 terms. So, there are 25 pairs of such terms.

25 × 561 = 14 025
The value of 11 + 22 + … + 539 + 550 is 14 025.

(A) 13 750 (B) 14 025 (C) 15 455
(D) 28 050 (E) None of the above

2. How many numbers at most can we select from

1, 2, 3, 4, 5, …, 46, 47, 48, 49 and 50

such that the sum of any two numbers is divisible by 5?

[Number Theory]

We can select all multiples of 5 that are from 1 to 50. The sum of any two multiples of 5 can be divided by 5.

50 ÷ 5 = 10
At most, 10 numbers can be selected.

(A) 5 (B) 10 (C) 15
(D) 20 (E) 25

3. Helen wants to cut a 500-centimetre ribbon into shorter pieces of length 30 centimetres or 80 centimetres without any length of ribbon left over. How many ways can she cut the ribbon? [Problem solving / Make a systematic list]

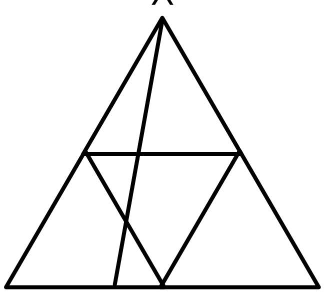
We can list the number of 80-cm pieces to be cut and check if the remaining length of ribbon is divisible by 30.

Number of 80-cm piece	Remaining length of ribbon	Check
1	420 cm	✓
2	340 cm	×
3	260 cm	×
4	180 cm	✓
5	100 cm	×

She can cut the ribbon in 2 ways.

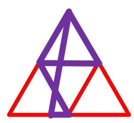
(A) 1 (B) 2 (C) 3
(D) 4 (E) 5

4. The figure shows a large triangle made up of 4 small equilateral triangles. Line XY is drawn across the figure. How many triangles are there in the figure? [Spatial Visualisation]

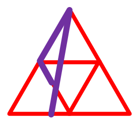


There are 1 large equilateral triangle and 4 small equilateral triangles.

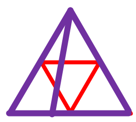
Count the smallest possible triangles along XY. There are 4 such triangles.



Then, count the triangles along XY that are made up of two triangles. There is 1 such triangle.



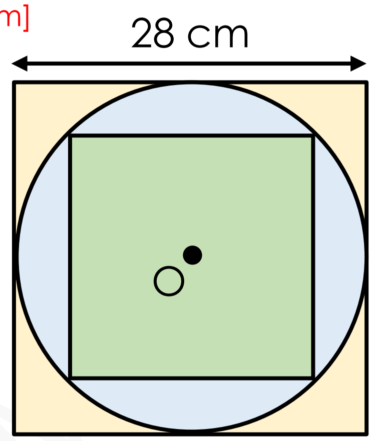
Then, count the triangles along XY that are made up of more than two triangles. There are 2 such triangles.



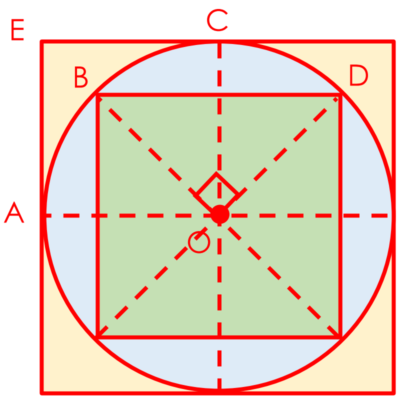
1 + 4 + 4 + 1 + 2 = 12
There are 12 triangles.

(A) 6 (B) 8 (C) 10
(D) 12 (E) None of the above

5. The figure below shows two squares and a circle. Point O is the centre of the figure. Find the area of the smaller square. [Mensuration / Simplify the problem]



We can draw some dotted lines to show that the area of the smaller square is half of the area of the larger square.



Observe that the larger square can be divided as shown into 4 identical squares of the same size as Square ECOA.

Observe that OA = OB = OC = OD because they are radii of the circle.

This also shows that the area of Triangle BOD is half of the area of Square ECOA.

Since the smaller square (green) can be divided into 4 identical triangles of the same size as Triangle BOD, the smaller square has an area half of the area of the larger square.

$28 \times 28 = 784$
The area of the larger square is 784 cm^2.

$784 \div 2 = 392$
The area of the smaller square is 392 cm^2.

(A) 196 cm^2 (B) 392 cm^2 (C) 490 cm^2
(D) 588 cm^2 (E) None of the above

Questions 6 to 10 carry 4 marks each.

6. Study the number pattern.

$$1 = 1 = \frac{1 \times 2}{2}$$
$$1 + 2 = 3 = \frac{2 \times 3}{2}$$
$$1 + 2 + 3 = 6 = \frac{3 \times 4}{2}$$
$$1 + 2 + 3 + 4 = 10 = \frac{4 \times 5}{2}$$
$$1 + 2 + 3 + 4 + 5 = 15 = \frac{5 \times 6}{2}$$
$$\vdots$$

Find the largest possible value of n such that
$1 + 2 + 3 + 4 + \ldots + n < 200$. [Patterns and sequences]

We need to find the product of two consecutive numbers that when divided by 2 is smaller than 200.

In other words, the product of the two consecutive numbers is smaller than 400.

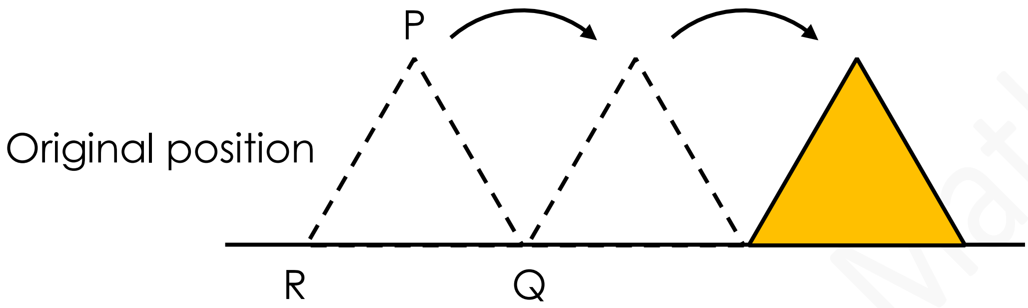
$20 \times 21 = 420$ which is greater than 400.
So, we can try 19×20.
$19 \times 20 = 380$ which is smaller than 400.

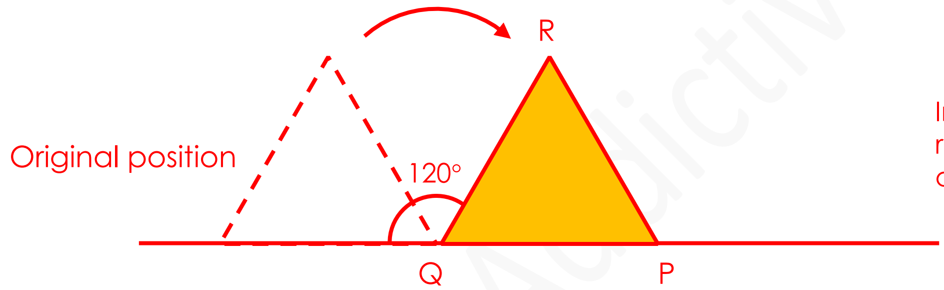
So, the largest possible value of n is 19.

(A) 13 (B) 14 (C) 19
(D) 20 (E) None of the above

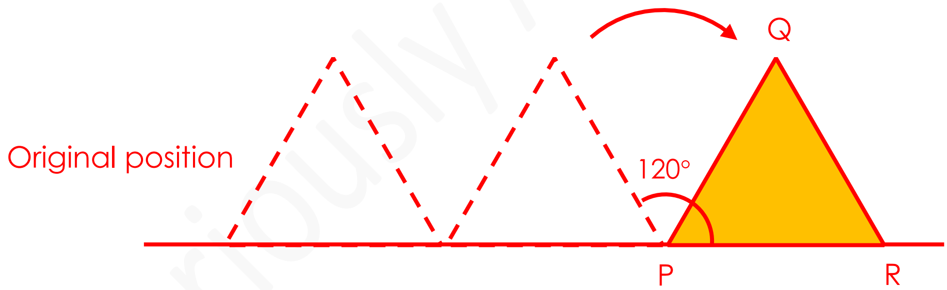
7. Ken placed the base of an equilateral triangular block PQR on flat ground. He rotated the block clockwise about a vertex twice as shown in the diagram. Find the total angle Point R rotated in the clockwise direction.

[Geometry]





In the first rotation, Point R rotated 120° in the clockwise direction.



In the second rotation, Point R rotated 120° in the clockwise direction.

120° + 120° = 240°
Point R rotated 240° in the clockwise direction.

(A) 120° (B) 180° (C) 240°

(D) 360° (E) None of the above

S.A.M Seriously Addictive Mathematics
Singapore Math

8. John is leaving his house to meet his friend at Town X. If he drives at an average speed of 80 km/h, he will be 20 minutes late. If he drives at an average speed of 100 km/h, he will be 10 minutes early. Find the distance between his house and Town X. [Problem solving]

Speed ratio

Early	:	Late
100		80
5		4

(÷ 20 on both sides)

Time ratio

Early	:	Late
4		5

20 + 10 = 30

The difference in duration is 30 min or $\frac{1}{2}$ h.

1 unit $= \frac{1}{2}$

4 units $= 4 \times \frac{1}{2}$

$= 2$

John will take 2 h to reach Town X if he drives at an average speed of 100 km/h.

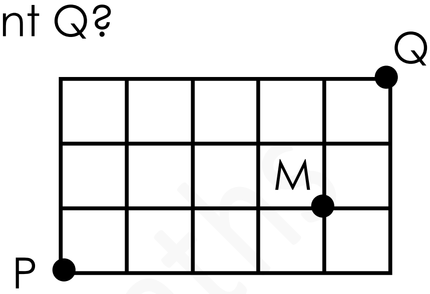
100 km/h × 2 h = 200 km

The distance between his house and Town X is 200 km.

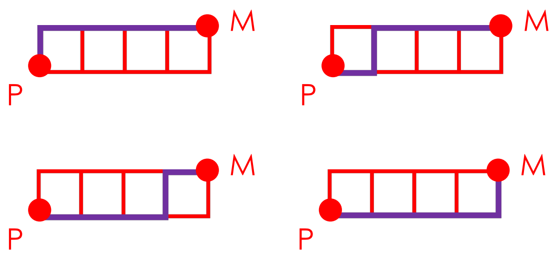
(A) 160 km (B) 200 km (C) 240 km

(D) 250 km (E) None of the above

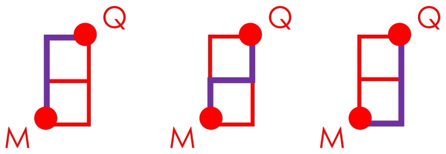
9. The lines in the diagram show the paths from Point P to Point Q. Joe wants to take the shortest path from Point P to Point Q passing through Point M. How many different ways can he move from Point P to Point Q?



[Combinatorics]



There are 5 different ways to move from Point P to Point M.



There are 3 different ways to move from Point M to Point Q.

$5 \times 3 = 15$
He can move from Point P to Point Q passing through Point M in 15 different ways.

(A) 8 (B) 10 (C) 12

(D) 14 (E) None of the above

10. There were five teams A, B, C, D and E in a football competition. Each team must play exactly once against another team.

[Logical reasoning]

a) Team A has played exactly 4 games.

b) Team B has played exactly 3 games.

c) Team C has played exactly 2 games.

d) Team D has played exactly 1 game.

Which of the following statements is **false**?

From **a)**, we know that Team A has played with each team because there are 4 opposing teams and each team only played exactly once against another team.

From **a)** and **d)**, we know that Team D has played 1 game and that was with Team A.

From **a)** and **c)**, we know that Team C has played 2 games and 1 of them was with Team A. The second game was with either Team B or E.

If the second game was with Team E, then Team B must have played with teams A, D and E, which is impossible because Team D only played with Team A.

Therefore, the second game Team C played was with Team B.

Since we know Team B played 3 games, one with Team A and one with Team C, the third game must be with Team E.

So, we have the following:
- Team A has played with teams B, C, D and E.
- Team B has played with teams A, C and E.
- Team C has played with teams A and B
- Team D has played with Team A.
- Team E has played with teams A and B.

(A) Team A has played with each team exactly once.
(B) Team B has played with Team E.
(C) Team C has played with Team B.
(D) Team D has played with Team A.
(E) Team E has played with Team C.

Section B

Questions 11 and 12 carry 6 marks each.

11. In Mathematics, we have the following:

$2^2 = 2 \times 2$

$2^3 = 2 \times 2 \times 2$

$2^4 = 2 \times 2 \times 2 \times 2$

$2^5 = 2 \times 2 \times 2 \times 2 \times 2$

Find the ones digit in the result of the following.

$$1^3 + 2^5 + 3^7 + 4^9 + 5^{11}$$

[Arithmetic]

The ones digit in 1^3 is 1.

$2^5 = 32$
The ones digit in 2^5 is 2.

3, 9, 27, 81, 243, 729, …
The ones digit follow the pattern 3, 9, 7, 1, 3, 9, 7, 1, ….
The ones digit in 3^7 is 7.

4, 16, 64, 256, 1024, …
The ones digit follow the pattern 4, 6, 4, 6, 4, 6, ….
The ones digit in 4^7 is 4.

5, 25, 125, …
The ones digit in 5^{11} is 5.

1 + 2 + 7 + 4 + 5 = 19
The ones digit in the result is 9.

12. In the following cryptarithm, each letter represents a different digit.

$$\begin{array}{rcccc} & M & A & T & H \\ \times & & & & 4 \\ \hline & H & T & A & M \end{array}$$

What 4-digit number does MATH represent? [Cryptarithm]

Observe that the product of a 4-digit number and 4 gives a 4-digit number.
So, the letter M can only be digit 1 or 2.
If M = 1, then H × 4 gives a product with the digit 1 in the ones place. This is not possible.
So, M = 2.

Since M = 2, H = 8.
Since 2 × 4 = 8, it means that there is no renaming when A × 4.
So, the possible digits for letter A are 0 or 1.

$$\begin{array}{rcccc} & 2 & A & \overset{3}{T} & 8 \\ \times & & & & 4 \\ \hline & 8 & T & A & 2 \end{array}$$

If A = 0, it means that T × 4 gives a product that has the digit 7 in the ones place. This is not possible. So, A = 1.

Since A = 1, then T × 4 is a product with the digit 8 in the ones place.
So, T = 7.

$$\begin{array}{rcccc} & 2 & \overset{3}{1} & \overset{3}{7} & 8 \\ \times & & & & 4 \\ \hline & 8 & 7 & 1 & 2 \end{array}$$

The 4-digit number MATH represents 2178.