P6 syllabus (Calculators Allowed)

Numeration
Number notation and place values, including decimals and fractions
Application of factors and multiples in problem solving
Numerical manipulations
Solving word problems involving the 4 operations, including order of operations

Algebra
Representation of an unknown number using a letter
Solving word problems involving algebraic expressions

Percentage
Solving for percentage part or whole of a quantity
Expressing one quantity as a percentage of another
Comparison of two quantities by percentage
Solving word problems involving percentage

Ratio
Expressing a ratio on its simplest form
Solving word problems involving ratio of two or three given quantities, their sum or difference, before and after change
Exclude ratios involving fractions and decimals

Measurement
Time in years, months, weeks, days, hours, minutes and seconds, 24-hour clock
Area and Perimeter of composite figures
Length, mass and volume
Area of triangle (exclude finding the base/height of a triangle given its area)
Volume of cube and cuboid
Area and circumference of circles
Spatial visualisation

Geometry
Properties of angles
Angles in composite figures
Nets

Data Analysis
Interpreting data from tables, bar charts, pie charts and line graphs
Average

Speed
Distance, Time and Speed
Solving word problems involving rate and speed

Logic
Inferring and deducing relationships between objects given a set of clues

Combinatorics
Finding largest, smallest or optimal number of objects satisfying certain criteria, deciding when the criteria can be met.

		Topics	Total marks	Question Numbers
1 to 10	2 marks	Numeration	15	22, 1, 17, 9
11 to 20	4 marks	Fractions, Decimals	14	21, 15, 11,
21	6 marks	Percentage	10	20, 14, 8
22	7 marks	Ratio	6	13, 7
23	8 marks	Measurement	6	2, 6
24	9 marks	Geometry	6	18, 5
25	10 marks	Data Analysis	11	4, 24
		Logic	6	12, 3
		Combinatorics	10	23, 10
		Algebra	6	16, 19
		Speed	10	25

P6

1. The sum of 10 consecutive odd numbers is 20 000. What is the smallest of these numbers?

Let the numbers be $(n - 8)$, $(n - 6)$, $(n - 4)$, $(n - 2)$, n, $(n + 2)$, $(n + 4)$, $(n + 6)$, $(n + 8)$, $(n + 10)$

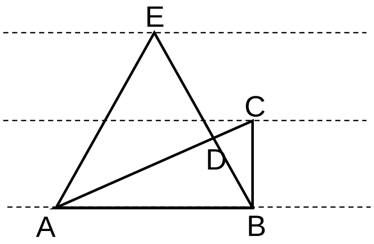
Sum of these numbers: $10n + 10 = 20\,000$

$10n = 19\,990$

$n = 1999$

The smallest number = $n - 8$ = <u>1991</u>

2. In the figure, the horizontal lines are equally spaced.
Triangle ABE is an equilateral triangle and triangle ABC is a right-angled triangle.



Which of the following statements about the difference in the areas of triangle ADE and triangle BCD is correct?

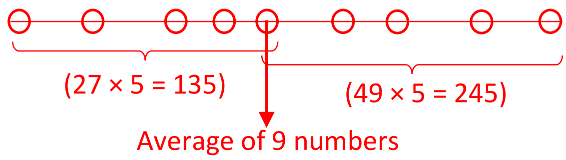
(A) The difference is equal to the area of triangle ABC.
(B) The difference is equal to the area of triangle ABD.
(C) The difference is 1.5 times the area of triangle ABC.
(D) The difference is twice the area of triangle ABD.

Let the area of triangle ABD be x and the area of triangle BCD be y.
The area of triangle ABC is thus x + y and the area of triangle ABE is 2x + 2y.
The area of triangle ADE is thus 2x + 2y – x = x + 2y.
The difference in the areas of triangle ADE and BCD
= x + 2y – y = x + y which is equal to the area of triangle ABC.
Answer: (<u>A</u>)

3. Sam has 3 pairs of blue boots and 4 pairs of black boots in a box. If Sam pulls out a single boot at a time without looking into the box, at least how many must he pull out to be sure to get a matching pair that he can put on?
(Note: Sam cannot put on boots which are both left-sided or both right-sided.)

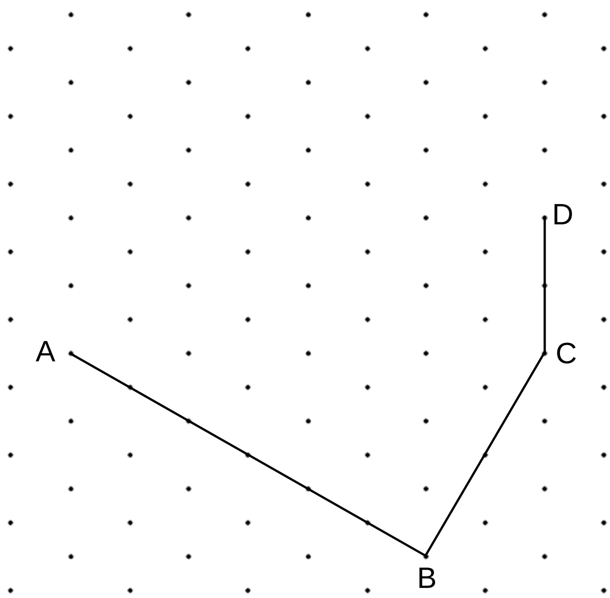
He has to pull out <u>8 boots</u> to be sure. Assuming that Sam has the misfortune of pulling out all 7 left-sided boots. The 8th boot will surely match any of the previous 7.

4. Fay wrote down nine numbers in increasing order. The middle number is the average of all nine numbers. The average of the first five is 27 and the average of the last five is 49. What is the sum of all the numbers?

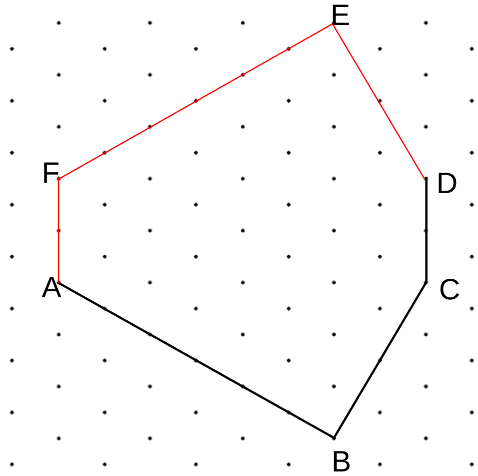


135 + 245 = 380
380 is the sum of ten numbers with the middle number counted twice.
The middle number is 380 ÷ 10 = 38
The sum of all the number is 380 – 38 = <u>342</u>

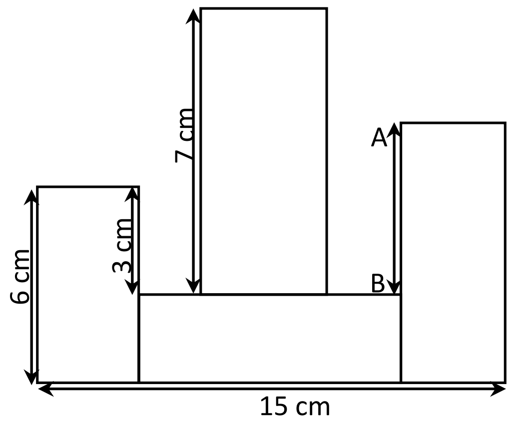
5. Jimmy wants to draw a 6-sided figure. He has drawn lines AB, BC and CD, where AB is perpendicular to BC. Continue drawing the figure such that ∠CDE is 150°, ∠DEF is 90°, and FA is parallel to DC. What is the value of ∠EFA?
(DO NOT USE A PROTRACTOR.)



∠EFA = 120°



6. The perimeter of the figure below, not drawn to scale, is 67 cm. Find the height of AB.



Sum of all vertical lengths = 67 – 15 – 15 = 37
Height of AB = (37 – 6 – 3 – 7 – 7 - 3) ÷ 2 = 5.5 cm

7. At a funfair, the ratio between the number of food stalls to the number of game stalls is 2 : 3 and the ratio of adult to children is 5:16. The ratio of the total number of stalls to the total number of people is 3 : 7. What is the ratio of the number of game stalls to the number of children?

Food stalls	Game stalls		Stalls	People		Adult	Children
2	3		3	7			
6	9		15	35		5	16
18	27		45	105		25	80

The ratio of the number of game stalls to the number of children is 27 : 80

8. If 3 dolls and 5 teddy bears cost as much as 5 dolls and 2 teddy bears, by what percentage is a doll more expensive than a teddy bear?

Cost of 3 dolls and 5 teddy bears = Cost of 5 dolls and 2 teddy bears
Cost of 3 teddy bears = Cost of 2 dolls
Cost of 1 doll = Cost of 1.5 teddy bears
If cost of 1 teddy bear = 100%, then cost of 1 doll = 150%
Cost of a doll is 50% more than the cost of a teddy bear.

9. The product of the two page numbers to which my book is opened is 6162.
What is the page number on the right-hand side?

The square root of 6162 is between 78 and 79. Hence, the two factors of 6162 which are consecutive numbers would be 78 and 79. The page number on the right-hand side is 79.

10. A group of children were separated into 2 unequal teams. Everyone within the teams shook hands with each other. If the total number of handshakes from both teams is 21, how many children were there altogether?

Make a list of possible number of handshakes.
2 people ….. 1 handshake
3 people ….. 3 handshakes
4 people ….. 6 handshakes
5 people ….. 10 handshakes
6 people ….. 15 handshakes
7 people ….. 21 handshakes

Since 21 handshakes = 15 handshakes + 6 handshakes, there are 6 + 4 = 10 children altogether.

11. Given that $\frac{1}{4-\frac{3}{2+\frac{1}{n}}} = \frac{5}{13}$, what is the value of n?

$$\frac{1}{4-\frac{3}{2+\frac{1}{n}}} = \frac{5}{13}$$

$$20 - \frac{15}{2+\frac{1}{n}} = 13$$

$$\frac{15}{2+\frac{1}{n}} = 7$$

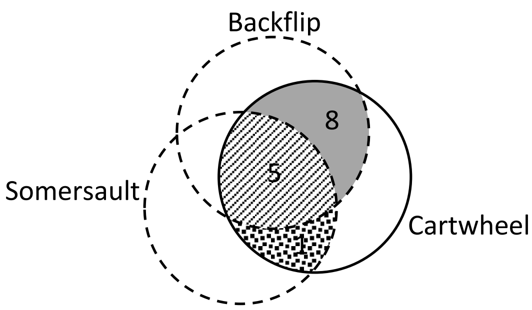
$$14 + \frac{7}{n} = 15$$

$$\frac{7}{n} = 1$$

$\underline{n = 7}$

12. In a group of young gymnasts, 24 of them can do the cartwheel, 13 of them can do the backflip and 6 of them can somersault. There are 8 gymnasts who can do the backflip but not the somersault. What is the least possible number gymnasts who can only do the cartwheel?

For the least possible number gymnasts who can only do the cartwheel, all the rest who can do both the backflip and the somersault should also be able to do the cartwheel. Since there are 8 gymnasts who can do the backflip but not the somersault, then the remaining 13 – 8 = 5 should be able to do either all three, and 6 – 5 = 1 can do both the somersault and the cartwheel. The least possible number who could only do the cartwheel is 24 – 8 – 5 – 1 = 10.



13. Every month, Vanessa spends a part of her salary and saves the rest. The ratio of the amount she spends to the amount she saves is 2 : 7. This month, she spent more than she usually does and the ratio became 3 : 7. By what percentage did she increase her spending?

Since the salary remains the same, the total number of units remains the same.

Hence,	usual ratio	→	20 : 70	(total = 90 units)
	this month's ratio	→	27 : 63	(total = 90 units)

Increased percentage spending = $\frac{7}{20} \times 100\%$ = <u>35%</u>

14. What amount of water should be added to reduce 200 ml of 5% sugar solution to 2% sugar solution?

Old solution: 5% of 200 ml → 10 ml
New solution: 2% → 10 ml
100% → 500 ml
Amount of water added = 500 – 200 = <u>300 ml</u>

15. A group of 22 scouts went on a trip. They prepared enough food to last 18 days. At the last minute, 14 additional scouts joined them. If they still want the food to last 18 days, what fraction of the daily portion should each scout eat per day?

Total amount of food = 22 × 18 = 396 units
New amount of food needed = 36 × 18 = 648 units
Amount of food short = 252 units
252 ÷ 36 = 7
Each scout is short of 7 units of food over 18 days.
Therefore, each scout should only eat $\frac{18-7}{18} = \frac{11}{18}$ of the daily portion per day.

16. Mr Ali's age is equal to the sum of the ages of his four children. His age h years ago, was twice the sum of their ages then. What is the ratio of Mr Ali's age in h years' time to the sum of his children's age in h years' time?

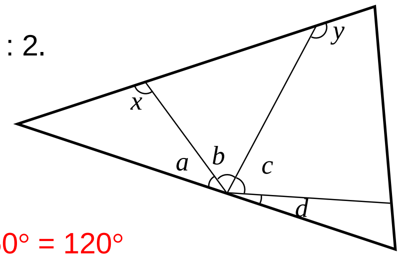
The ratio of their ages in h years' time is <u>8 : 11</u>

17. Given that $1001^1 = 1001$
$1001^2 = 1001 \times 1001 = 1\,002\,001$
$1001^3 = 1001 \times 1001 \times 1001 = 1\,003\,003\,001$,and so on,

What is the sum of all the digits in the answer for 1001^{11} ?

The digit sum for 1001^1 is 2
1001^2 is $4 = 2^2$
1001^3 is $8 = 2^3$ and so on
Therefore, the digit sum for 1001^{11} is 2^{11} which is <u>2048</u>

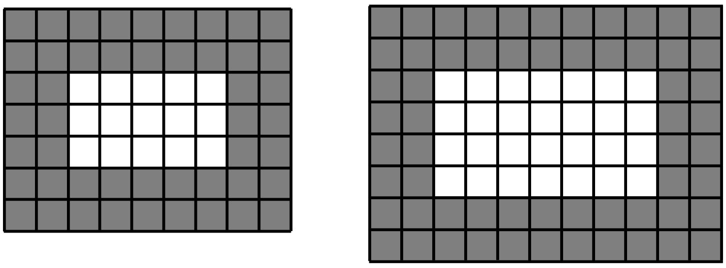
18. The values of $\angle a$, $\angle b$, $\angle c$ and $\angle d$ are in the ratio 3 : 5 : 5 : 2.
What is the value of $\angle x + \angle y$?



$\angle b = \frac{5}{15} \times 180° = 60°$

The sum of the two angles adjacent to $\angle x$ and $\angle y = 180° – 60° = 120°$
Therefore, $\angle x + \angle y = 360° – 120° = 240°$

19. Tom arranges some white tiles into rectangular shapes. Paul then surrounds the shapes with 2 layers of grey tiles. Below are two examples:



How many tiles will Paul use if Tom forms a rectangle with x tiles along its length and y tiles along its breadth? Give your answer in x and y.

Number of grey tiles = $4 \times (x + y + 4) =$ <u>$4x + 4y + 16$</u>

20. A tree increases its number of fruits at the rate of 50% every year. What was the number of fruits produced by the tree 3 years ago, if this year it produced 54 fruits?

This year: 150% $\rightarrow$ 54
Last year: 100% $\rightarrow \frac{100}{150} \times 54$
2 years back: $\frac{100}{150} \times \frac{100}{150} \times 54$
3 years back: $\frac{100}{150} \times \frac{100}{150} \times \frac{100}{150} \times 54 =$ <u>16 fruits</u>

21. Given that $\frac{1}{a} - \frac{1}{b} = \frac{1}{y}$ where a and b are consecutive numbers;

and that $\frac{1}{x} - \frac{1}{y} = \frac{1}{3080}$, where x and y are consecutive numbers, find the value of a.

If $\frac{1}{a} - \frac{1}{b} = \frac{1}{y}$, then $y = a \times b$ where a and b are consecutive numbers

If $\frac{1}{x} - \frac{1}{y} = \frac{1}{3080}$, then $3080 = x \times y$ where x and y are consecutive numbers

Since √(3080) is between 55 and 56, then 3080 = 55 × 56 and $y = 56$
Since $y = a \times b$, and 56 = 7 × 8, then $\underline{a = 7}$

22. Which 3-digit number has exactly five factors including 1 and itself?

When a number has an odd number of factors, it is a square number.
When a number has 3 factors, it is the square of a prime number.
When a number has 5 factors, it is the square of the square of a prime number.
The only 3-digit number that is the square of the square of a prime number is 625.

23a) How many 3-digit numbers can be formed from the digits 3, 4, 5, 6, 7 and 8 with no repetition of digits? (1 mark)

6 × 5 × 4 = 120 numbers

b) How many ways can 6 people be divided into 2 equal groups? (2 mark)

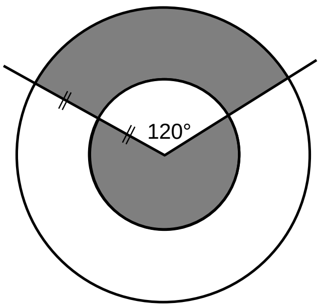
$\frac{6\times5\times4}{3\times2\times1}$ = 20 ways

c) Daniel has 6 pieces of straws of different lengths. The lengths are 5 cm, 6 cm, 8 cm, 9 cm, 11 cm and 15 cm. How many different triangles can he form by joining three straws end to end? (5 marks)

A triangle can only be formed when the sum of the two shorter sides is longer than the longest side. Therefore, there are only 14 different triangles that can be formed.

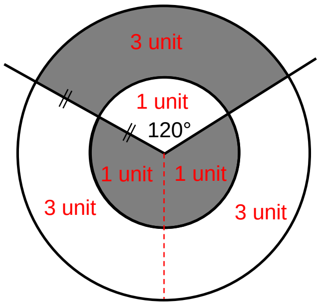
The triangles are:
5 cm by 6 cm by 8 cm
5 cm by 6 cm by 9 cm
5 cm by 8 cm by 9 cm
5 cm by 8 cm by 11 cm
5 cm by 9 cm by 11 cm
5 cm by 11 cm by 15 cm
6 cm by 8 cm by 9 cm
6 cm by 8 cm by 11 cm
6 cm by 9 cm by 11 cm
6 cm by 11 cm by 15 cm
8 cm by 9 cm by 11 cm
8 cm by 9 cm by 15 cm
8 cm by 11 cm by 15 cm
9 cm by 11 cm by 15 cm

24. Both the white and grey parts of the 120° sector of the Pie-Chart represent the number of pupils from Primary 1 to Primary 2. The white and grey parts of the remaining sector represents the number of pupils from Primary 3 to Primary 6. The white parts of the Pie-Chart represent those who like Maths and the grey parts represent those who dislike Maths. Find the ratio of all the pupils who like Maths to those who dislike Maths.



Comparing the two circles:
Since the radius of the larger circle is twice the radius of the smaller circle, then the area of the larger circle is 4 times the area of the smaller circle.
Hence, the figure can be divided into the following parts:

Grey parts → 5 units
White parts → 7 units



Ratio of pupils who like Maths to those who do not → <u>7 : 5</u>

25. Two trains each 400 m long, pass each other completely in 10 seconds when they are moving in opposite direction. Moving in the same direction, they pass each other completely in 20 seconds. Find the speed of the faster train.

For both trains to completely pass each other travelling in opposite directions, they would have travelled a total distance of 400 m + 400 m = 800 m

$10 \times (v_1 + v_2) = 800$

$v_1 + v_2 = 80$

For both trains to completely pass each other travelling in the same direction:

$20 \times (v_2 - v_1) = 400$

$v_2 - v_1 = 20$

Adding the two equations, $v_1 + v_2 + v_2 - v_1 = 80 + 20$

$2v_2 = 100$

$v_2 = 50$ m/s

The speed of the faster train is 50 m/s.